Physics Beyond the Standard Model

Exercise sheet 6 - Solutions

Exercise 1: Wess-Zumino model

$$
\begin{aligned}
\int \Phi^\dagger \Phi d^2\theta d^2\bar{\theta} &= (\partial_\mu \varphi^*)(\partial^\mu \varphi) - \frac{i}{2}(\bar{\psi}\tilde{\sigma}^\mu \partial_\mu \psi + \psi \sigma^\mu \partial_\mu \bar{\psi}) + F^* F \\
\Phi^2 &= \varphi^2 + 2\sqrt{2}\varphi(\theta\psi) + (\theta\theta)(2\varphi F - \psi\psi) \\
\Phi^3 &= \varphi^3 + 3\sqrt{2}\varphi^2(\theta\psi) + 3(\theta\theta)(\varphi^2 F - \varphi\psi\psi) \\
\int W(\Phi) d^2\theta &= F\left(m\varphi + \frac{\lambda}{2}\varphi^2\right) - \left(\frac{m}{2} + \frac{\lambda}{2}\varphi\right)\psi\psi \\
\int W(\Phi^\dagger) d^2\bar{\theta} &= F^*\left(m\varphi^* + \frac{\lambda}{2}\varphi^{*2}\right) - \left(\frac{m}{2} + \frac{\lambda}{2}\varphi^*\right)\bar{\psi}\bar{\psi}
\end{aligned}
$$

$$
\Longrightarrow \boxed{
\begin{aligned}
\mathcal{L} &= (\partial_\mu \varphi^*)(\partial^\mu \varphi) - \frac{i}{2}(\bar{\psi}\tilde{\sigma}^\mu \partial_\mu \psi + \psi \sigma^\mu \partial_\mu \bar{\psi}) \\
&\quad + \frac{m}{2}(\psi\psi + \bar{\psi}\bar{\psi}) + \frac{\lambda}{2}(\varphi\psi\psi + \varphi^*\bar{\psi}\bar{\psi}) \\
&\quad + F^* F - (m\varphi + \frac{\lambda}{2}\varphi^2)F - (m\varphi^* + \frac{\lambda}{2}\varphi^{*2})F^*
\end{aligned}
}
$$

We eliminate the auxiliary fields using the Euler-Lagrange equation:

$$
\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial F^*} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu F^*)} \stackrel{!}{=} 0 = \frac{\partial \mathcal{L}}{\partial F^*} = F - m\varphi^* - \frac{\lambda}{2}\varphi^{*2} \\
&\frac{\partial \mathcal{L}}{\partial F} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu F)} \stackrel{!}{=} 0 = \frac{\partial \mathcal{L}}{\partial F} = F^* - m\varphi - \frac{\lambda}{2}\varphi^2 \\
&\Rightarrow F = m\varphi^* + \frac{\lambda}{2}\varphi^{*2} \quad \& \quad F^* = m\varphi + \frac{\lambda}{2}\varphi^2
\end{aligned}
$$

The equations of motion of both auxiliary fields do not contain derivatives. Therefore, they don't describe propagation in space-time and have no physical meaning. By using the Euler-Lagrange equation one can therefore eliminate auxiliary fields and obtain a Langrangian independent of F and F^*:

$$
\boxed{
\begin{aligned}
\mathcal{L} &= (\partial_\mu \varphi^*)(\partial^\mu \varphi) - m^2|\varphi|^2 \\
&\quad - \frac{i}{2}(\bar{\psi}\tilde{\sigma}^\mu \partial_\mu \psi + \psi \sigma^\mu \partial_\mu \bar{\psi}) + \frac{m}{2}(\psi\psi + \bar{\psi}\bar{\psi}) \\
&\quad + \frac{\lambda}{2}(\varphi\psi\psi + \varphi^*\bar{\psi}\bar{\psi}) - m\frac{\lambda}{2}|\varphi|^2(\varphi + \varphi^*) - \frac{\lambda^2}{4}|\varphi|^4
\end{aligned}
}
$$

1st line: kinetic term of the bosonic field φ (= slepton,squark) of mass m
2nd line: kinetic term of the sermonic field ψ (= lepton,quark) of mass m
3rd line: Yukawa interaction and self-interaction of the bosonic field.
The masses of the particles and super particles are identical. Furthermore, the couplings of the Yukawa interaction and self-interaction of the scalars are universal. There is no relation between these couplings in the SM.
The SUSY transformation of the bosonic and fermionic fields is:

$$\boxed{\begin{array}{rcl} \delta\varphi & = & \sqrt{2}\varepsilon\psi \\ \delta\psi & = & i\sqrt{2}\sigma^{\mu}\bar{\varepsilon}\partial_{\mu}\varphi + \sqrt{2}\varepsilon m\varphi^{*} + \sqrt{2}\varepsilon\frac{\lambda}{2}\varphi^{*2} \end{array}}$$

Exercise 2: Radiative symmetry breaking

$$\begin{array}{rcl} \dfrac{\partial\overrightarrow{M}}{\partial t} & = & \dfrac{\alpha_t}{\pi}A\overrightarrow{M} \quad \text{with} \quad t \equiv \log\left(\dfrac{Q^2}{M_{GUT}^2}\right) \quad \& \quad A \equiv \begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \\ & & \overrightarrow{M} = \begin{pmatrix} M_{H_u}^2 \\ M_{\tilde{t}_R}^2 \\ M_{\tilde{t}_L}^2 \end{pmatrix} \quad \& \quad \overrightarrow{M}_0 = M_0^2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\ \text{Solution : } \overrightarrow{M} & = & \exp\left(\dfrac{\alpha_t}{\pi}At\right)\overrightarrow{M}_0 = \displaystyle\sum_{n=0}^{\infty}\left(\frac{\alpha_t}{\pi}t\right)^n\frac{A^n}{n!}\overrightarrow{M}_0 \\ A^2 & = & 6A \implies A^n = 6^{n-1}A \\ \implies \overrightarrow{M} & = & \left[\mathbb{1} + \displaystyle\sum_{n\geq 1}\left(\frac{\alpha_t}{\pi}t\right)^n\frac{A^n}{n!}\right]\overrightarrow{M}_0 = \left\{\mathbb{1} + \frac{1}{6}\left[\exp\left(\frac{6\alpha_t}{\pi}t\right) - 1\right]A\right\}\overrightarrow{M}_0 \\ & & \text{with } A\overrightarrow{M}_0 = 3M_0^2\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \end{array}$$

$$\begin{array}{rcl} M_{H_u}^2 & = & \left\{1 + \dfrac{9}{6}\left[\exp\left(\dfrac{6\alpha_t}{\pi}t\right) - 1\right]\right\}M_0^2 = \dfrac{1}{2}\left[3\exp\left(\dfrac{6\alpha_t}{\pi}t\right) - 1\right]M_0^2 \\ M_{H_u}^2 & < & 0 \Leftrightarrow 3\exp\left(\dfrac{6\alpha_t}{\pi}t\right) - 1 < 0 \quad \text{for} \quad t(Q^2 = M_Z^2) = \log\left(\dfrac{M_Z^2}{M_{GUT}^2}\right) \\ & & \Leftrightarrow \dfrac{6G_F m_t^2}{\sqrt{2}\pi^2}\log\left(\dfrac{M_{GUT}^2}{M_Z^2}\right) > \log 3 \\ & & \Leftrightarrow m_t^2 > \dfrac{\sqrt{2}\pi^2}{6G_F}\dfrac{\log 3}{\log\left(\frac{M_{GUT}^2}{M_Z^2}\right)} \approx 3400\ GeV^2 \Rightarrow m_t > 58.3\ GeV \end{array}$$

Spontaneous symmetry breaking occurs naturally for large top masses.

$$
\begin{aligned}
M^2_{\tilde{t}_R} &= \left\{1 + \frac{6}{6}\left[exp\left(\frac{6\alpha_t}{\pi}t\right) - 1\right]\right\} M_0^2 = exp\left(\frac{6\alpha_t}{\pi}t\right) M_0^2 \; > \; 0 \\
M^2_{\tilde{t}_L} &= \left\{1 + \frac{1}{2}\left[exp\left(\frac{6\alpha_t}{\pi}t\right) - 1\right]\right\} M_0^2 = \frac{1}{2}\left[exp\left(\frac{6\alpha_t}{\pi}t\right) + 1\right] M_0^2 \; > \; 0
\end{aligned}
$$

The stop masses are positive $\forall t$.