PHYSICS BEYOND THE STANDARD MODEL

Exercise sheet 7

Exercise 1: Sfermions

The sfermion mass matrix in the MSSM is given by

$$\mathcal{L}_{\tilde{f}} = -\tilde{f}^\dagger M_{\tilde{f}}^2 \tilde{f} \qquad \text{with} \qquad \tilde{f} = \begin{bmatrix} \tilde{f}_L \\ \tilde{f}_R \end{bmatrix}$$

$$M_{\tilde{f}}^2 = \begin{bmatrix} m_f^2 + M_{\tilde{f}_L}^2 + M_Z^2(\frac{1}{2} - e_f s_W^2)\cos 2\beta & m_f(A_f - \mu r_f) \\ m_f(A_f - \mu r_f) & m_f^2 + M_{\tilde{f}_R}^2 + M_Z^2 e_f s_W^2 \cos 2\beta \end{bmatrix}$$

$$\text{with} \quad r_f = \begin{cases} \cot\beta & \text{for up-type fermions} \\ \tan\beta & \text{for down-type fermions} \end{cases}$$

Determine the mass eigenvalues and eigenstates.

Exercise 2: Supersymmetry breaking in quantum mechanics

The goal of this exercise is to construct a simple supersymmetric model and observe symmetry breaking. Consider a quantum mechanical system with commuting (x, y: bosonic) and anti-commuting (ξ, η: fermionic) coordinates:

$$\mathcal{L} = \frac{1}{2}\dot{\mathrm{x}}^2 - \mathrm{V(x)} + \eta\dot{\xi} + \mathrm{W(x)}\eta\xi \tag{1}$$

The quantisation is given by the following relations:

$$[\hat{p}, \hat{x}] = i\hbar \qquad \{\hat{\eta}, \hat{\xi}\} = \hbar \qquad (\hat{\eta}^2 = \hat{\xi}^2 = 0) \tag{2}$$

Consequenlty, the hamiltonian is:

$$H = \frac{1}{2}\hat{p}^2 + V(\hat{x}) - \frac{1}{2}W(\hat{x})[\hat{\eta}, \hat{\xi}] \tag{3}$$

Consider the following representation of the fermionic operators:

$$\hat{\xi} = \sigma_+ \text{ and } \hat{\eta} = \hbar\ \sigma_- \tag{4}$$

The wave function is a two-component Pauli spinor:

$$\psi(x,t) = \begin{pmatrix} \phi_1(x,t) \\ \phi_2(x,t) \end{pmatrix} \tag{5}$$

where ϕ_1 is the bosonic and ϕ_2 the fermionic component.

a) Express the Hamiltonian using Pauli matrices.

b) What is the operator $N_F = \hat{\eta}\hat{\xi}$?

Consider now the following potentials:

$$V(\hat{x}) = \frac{1}{2}v^2(\hat{x}) \qquad \text{and} \qquad W(\hat{x}) = \frac{\partial v(\hat{x})}{\partial x} = v'(\hat{x}) \tag{6}$$

with the operators

$$Q_i = \frac{1}{2}\sigma_i \left[\hat{p} + i\sigma_3 v(\hat{x})\right] \qquad \text{where} \quad i = 1,2 \tag{7}$$

c) Calculate $[H, Q_i]$ and $\{Q_i, Q_j\}$. What do these relations mean?

d) Rewrite the Q_i in matrix notation. How do they act on $\psi(x,t)$?

e) Consider the vacuum $|0\rangle$, with $H|0\rangle = 0$. What do you find for $Q_i|0\rangle$?

f) Show that $|f\rangle = -\sqrt{\frac{2}{E}}Q_1|b\rangle = i\sqrt{\frac{2}{E}}Q_2|b\rangle$ is a fermionic state. What is the corresponding bosonic state?

g) What happens to SUSY if the vacuum state has positive energy?