Physics Beyond the Standard Model

Exercise Sheet 6

Exercise 1: Wess-Zumino Model

We can extend the action derived in exercise sheet 5 by a superpotential term:

$$S = \int \Phi^\dagger \Phi d^4x d^2\theta d^2\bar{\theta} - \int W(\Phi) d^4x d^2\theta - \int W(\Phi^\dagger) d^4x d^2\bar{\theta} \tag{1}$$

We use the result from sheet 5 for the $\int \Phi^\dagger \Phi d^4x d^2\theta d^2\bar{\theta}$ term:

$$\mathcal{L}_{kin} = (\partial_\mu \varphi^*)(\partial^\mu \varphi) - \frac{i}{2}(\bar{\psi}\tilde{\sigma}^\mu \partial_\mu \psi + \psi \sigma^\mu \partial_\mu \bar{\psi}) + F^* F \tag{2}$$

a) What do we find for S if we make the ansatz:

$$W(\Phi) = \frac{m}{2}\Phi^2 + \frac{\lambda}{6}\Phi^3 \tag{3}$$

b) What to we obtain for F from the equations of motion?

c) Why can we eliminate F from $\mathcal{L}$? What do we obtain for $\mathcal{L}$ after eliminating F? What are the masses and couplings of the fields φ und ψ?

Exercise 2: Radiative Symmetry Breaking

Consider the following simplified ($A_t = 0,\ \mu = 0$) renormalisation group equation for and up-type Higgs mass and the two stop masses:

$$\frac{\partial}{\partial\left(\log Q^2/M_{\mathrm{GUT}}^2\right)} \begin{bmatrix} M_{H_u}^2 \\ M_{\tilde{t}_R}^2 \\ M_{\tilde{t}_L}^2 \end{bmatrix} = \frac{\alpha_t}{\pi} \begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} M_{H_u}^2 \\ M_{\tilde{t}_R}^2 \\ M_{\tilde{t}_L}^2 \end{bmatrix}$$

with $\alpha_t = G_F m_t^2/\sqrt{2}\pi$. Solve these equations under the constraint $M_{H_u} = M_{\tilde{t}_R} = M_{\tilde{t}_L} = M_0$ for $Q = M_{\mathrm{GUT}}$. Show that $M_{H_u}^2$ is negative for $Q = M_Z$ if the top mass is sufficiently large, while the mass squares of the stops remain positive.