Physics Beyond the Standard Model

# Exercise Sheet 5

**Exercise 1:** Chiral Multiplet

a) Consider the complex superfield $\Phi(x,\theta) = \varphi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x)$. The fields $(\varphi(x), \psi(x), F(x))$ are called the chiral multiplet.
The SUSY transformations are (with $\varepsilon$ infinitesimal):

$$\theta^\alpha \rightarrow \theta^\alpha + \varepsilon^\alpha \qquad \alpha = 1,2 \tag{1}$$
$$x^\mu \rightarrow x^\mu + i\theta\sigma^\mu\varepsilon \qquad \mu = 0,1,2,3 \tag{2}$$

How do $\varphi$, $\psi$ and $F$ transform under SUSY? Total derivatives can be rewritten as vanishing surface terms. Which terms can therefore appear in a Lagragian/action for $\Phi$?

b) The simplified representation in a) without $\bar{\theta}$ was possible because the field is chiral. The regular coordinates are recovered using $x^\mu \rightarrow x^\mu + i\theta\sigma^\mu\bar{\theta}$. Show that

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) = \varphi(x) + \sqrt{2}\theta\psi(x) + (\theta\theta)F(x) + i(\theta\sigma^\mu\bar{\theta})\partial_\mu\varphi(x) \\ -\frac{i}{\sqrt{2}}(\theta\theta)\partial_\mu(\psi(x)\sigma^\mu\bar{\theta}) - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box\varphi(x),\end{aligned} \tag{3}$$

holds by using $(\theta\sigma^\mu\bar{\theta})(\theta\sigma^\nu\bar{\theta}) = \frac{1}{2}g^{\mu\nu}(\theta\theta)(\bar{\theta}\bar{\theta})$ ($g^{\mu\nu}$ is the metric) and $(\theta\sigma^\mu\bar{\theta})(\theta\psi) = -\frac{1}{2}(\theta\theta)(\psi\sigma^\mu\bar{\theta})$.

c) The combination of chiral and anti-chiral superfields again yields a superfield of the same kind. The multiplication of a chiral and an antichiral super field yields the kinetic part of the Lagrangian. Calculate

$$\bar{\Phi}(x,\theta,\bar{\theta})\Phi(x,\theta,\bar{\theta}) \mid_{\theta\theta\bar{\theta}\bar{\theta}} . \tag{4}$$

**Exercise 2:** SUSY–QED

In the superfield formalism the mass terms and interaction terms can be derived from the renormalizable SUSY invariant superpotential:

$$W(\Phi) = \frac{m}{2}\Phi^2 + \frac{\lambda}{6}\Phi^3. \tag{5}$$

Show that

$$\begin{aligned} S_W &= \int d^6 z W(\Phi) + h.c. \\ &= \int d^4 x \left\{ F\frac{\partial W(\varphi)}{\partial \varphi} - \frac{1}{2}\frac{\partial^2 W(\varphi)}{\partial \varphi^2}\bar{\Psi}_L{}^c \Psi_L \right\} + h.c., \end{aligned}$$

where

$$\Phi(y,\theta) = \varphi(y) + \sqrt{2}\theta\psi(y) + (\theta\theta)F(y)\big|_{y=x+i\theta\sigma\bar{\theta}}$$