Physics Beyond The Standard Model

# Exercise sheet 3

**Exercise 1:** $SU(5)$ breaking

We consider the consecutive spontaneous symmetry breaking of the $SU(5)$:

$$SU(5) \xrightarrow{\phi} SU(3) \times SU(2) \times U(1) \xrightarrow{H} SU(3) \times U(1) \tag{1}$$

Here $H$ is a Higgs multiplet in vector representation and $\phi$ is another Higgs multiplet in the adjoint representation. The Higgs potential for the spontaneous breaking of the $SU(5)$ is given by

$$V(\phi) = M_{24}^2 Tr(\phi \cdot \phi^\dagger) + \lambda_{24} \left(Tr(\phi \cdot \phi^\dagger)\right)^2 + \lambda'_{24} Tr((\phi \cdot \phi^\dagger)^2) \tag{2}$$

To find a minimum of the potential we make the following ansatz:

$$\phi_0 = \langle \phi \rangle = \frac{v_{24}}{\sqrt{15}} diag(1, 1, 1, -3/2, -3/2) \tag{3}$$

Consider $M_{24}^2 < 0$, what does this mean for $\lambda_{24}$, $\lambda'_{24}$ and $v_{24}$?

The covariant derivative is:

$$D_\mu \phi = \partial_\mu \phi + ig[V_\mu, \phi] \tag{4}$$

Recall the corresponding terms in the Standard Model and determine $V_\mu$ in a similar fashion. Was does this mean for the masses of the 24 gauge bosons of the $SU(5)$.

The mass term from $tr|D_\mu \phi|^2$ has the usual form: $g^2 tr|[V_\mu, \phi_0]|^2$. What follows for $M_X, M_Y$?

**Exercise 2:** Running of the coupling constants and unification

The one-loop running for the coupling constants is given by (RGE):

$$\frac{1}{\alpha_j(Q^2)} = \frac{1}{\alpha_j(\mu^2)} + \frac{b_j}{4\pi} ln\left(\frac{Q^2}{\mu^2}\right) \tag{5}$$

In GUTs one finds for the $b_j$:

$$\begin{pmatrix} b1 \\ b2 \\ b3 \end{pmatrix} = \begin{pmatrix} 0 \\ 22/3 \\ 11 \end{pmatrix} - N_F \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} - N_H \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}, \tag{6}$$

where $N_F$ is the number of families and $N_H$ is the number of Higgs doublets.

1. Use the RGE given above to determine the unification scale by comparing $\alpha_1$ and $\alpha_2$, starting from the mass of the Z boson.

2. Determine the Weinberg angle from the Z boson mass as a function of $\alpha(m_Z)$ and $\alpha_s(m_Z)$.

Input parameters:

$$\begin{aligned} \sin^2\theta_w(M_{GUT}) &= \frac{3}{8} \\ m_Z &= 91.188\ GeV \\ \alpha(m_Z) &= \frac{1}{127.9} \\ \alpha_s(m_Z) &= 0.119 \pm 0.004 \end{aligned}$$