Physics Beyond The Standard Model

Exercise sheet 2

Aufgabe 1: QED–Lagrangian

Derive the corresponding equations of motion from the following Lagrangians,

$$\begin{aligned}\mathcal{L} &= |D_\mu\phi|^2 - m^2|\phi|^2 \\ \mathcal{L} &= \bar{\psi}(i\not{D} - m)\psi \\ \mathcal{L} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu\end{aligned}$$

using the covariant derivative

$$D_\mu = \partial_\mu + ieA_\mu\mathcal{Q}.$$

Aufgabe 2: Gauge boson masses

Show that in the Standard Model the Higgs field ϕ generates masses for the $W^\pm$ and Z gauge bosons but that the photon A remains massless. Proceed as follows:

1. Use the interaction term in the Lagrangian:

$$|D_\mu\phi_0|^2 = \left|\left(igW_\mu^a\frac{\sigma^a}{2} + ig'B_\mu\frac{Y}{2}\right)\begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix}\right|^2$$

2. Show that it can be written as:

$$|D_\mu\phi_0|^2 = \frac{v^2}{8}\begin{pmatrix}W_\mu^1\\ W_\mu^2\\ W_\mu^3\\ B_\mu\end{pmatrix}\begin{pmatrix}g^2 & 0 & 0 & 0\\ 0 & g^2 & 0 & 0\\ 0 & 0 & g^2 & -gg'\\ 0 & 0 & -gg' & g'^2\end{pmatrix}\begin{pmatrix}W_\mu^1\\ W_\mu^2\\ W_\mu^3\\ B_\mu\end{pmatrix}$$

3. Calculate the eigenvalues of the mass matrix and show that $M_W = \frac{1}{2}gv$, $M_Z = \frac{1}{2}\sqrt{g^2+g'^2}\,v$ and $M_A = 0$.