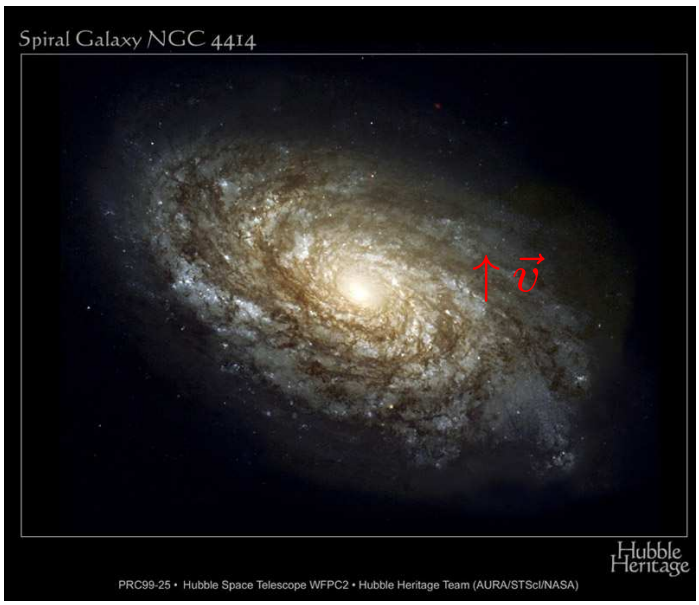
§9. Supersymmetric Dark Matter

- Why Dark Matter?

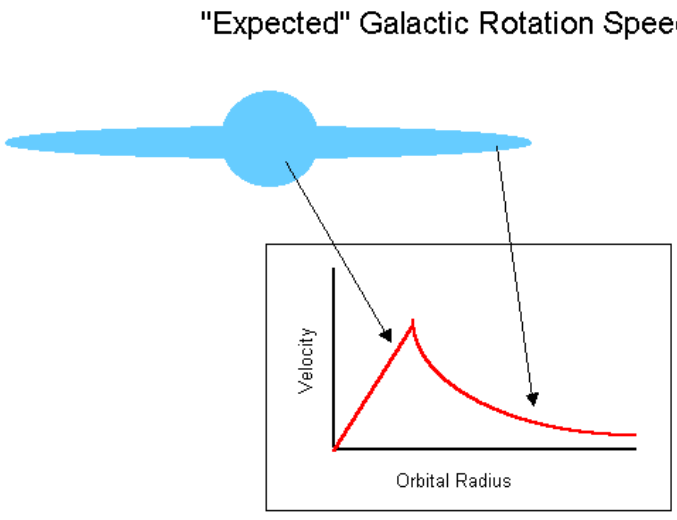
(i) spiral galaxies:

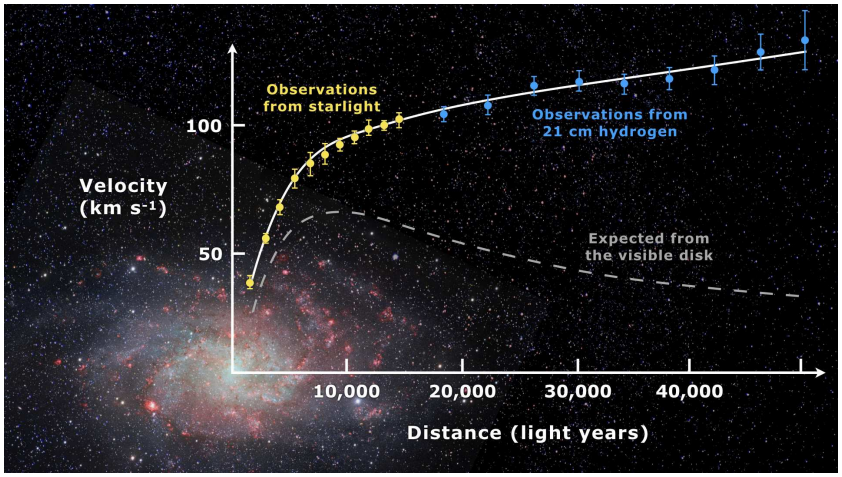


$$\frac{G_N M(r)}{r^2} \propto \frac{v^2}{r}$$

$$\Rightarrow r > r_{core}: \; v^2 \propto \frac{G_N M_{tot}}{r}$$

rotation curves: 21cm emission line of neutral hydrogen





$\Rightarrow$ Dark Matter

(ii) cosmic microwave background (CMB):

Friedmann equation: [homogeneous universe]

$$\boxed{H^2 = \frac{\dot{R}^2}{R^2} = \frac{8\pi G_N}{3}\rho - \frac{k}{R^2} + \frac{\Lambda}{3}}$$

H : Hubble constant
ρ : matter density
k : 3-dimensional curvature
Λ : cosmological constant

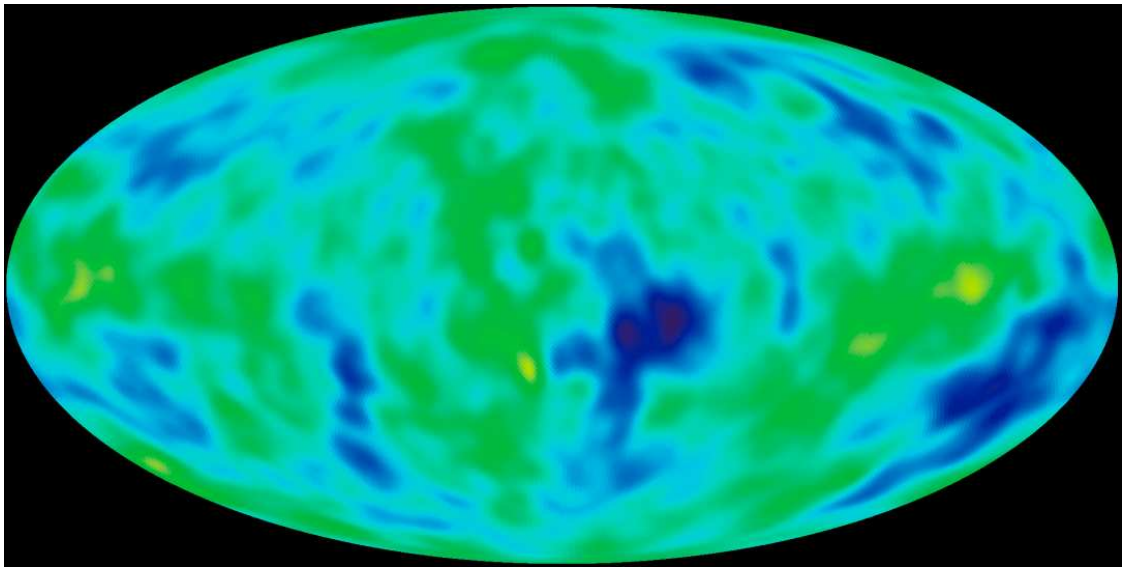
$$k = \begin{cases} 0 & \text{flat metric} \\ > 0 & \text{closed universe} \\ < 0 & \text{open universe} \end{cases}$$

$$k = \Lambda = 0 \quad \Rightarrow \quad \rho = \rho_c = \frac{3H^2}{8\pi G_N} \quad \text{critical density}$$

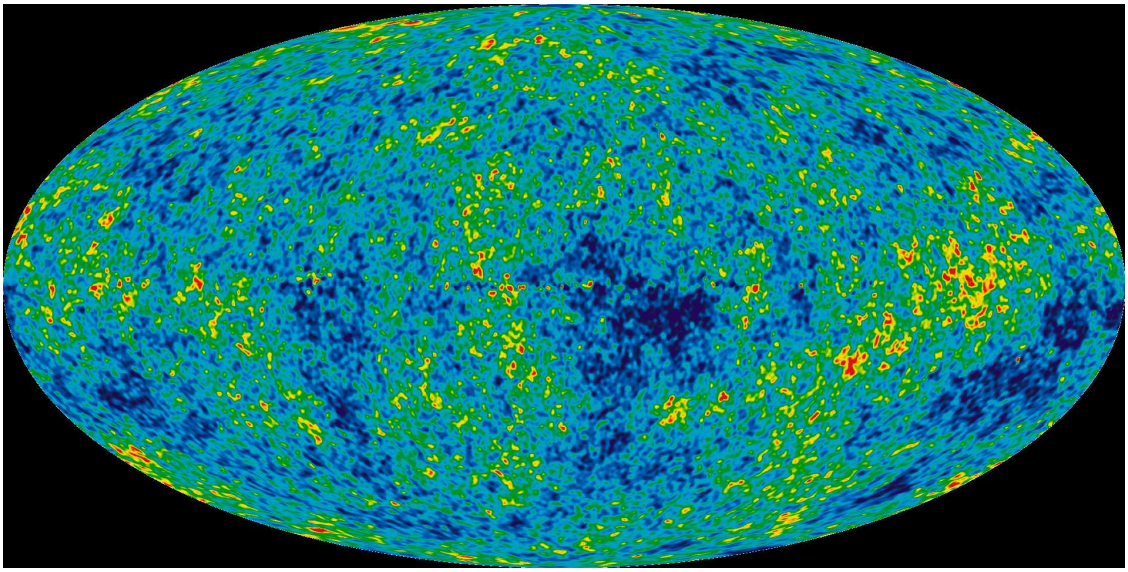
Friedmann equation:

$$\begin{aligned} (\Omega - 1)H^2 &= \frac{k}{R^2} \\ \Omega &= \Omega_m + \Omega_\Lambda \\ \Omega_m &= \frac{\rho}{\rho_c} \\ \Omega_\Lambda &= \frac{\Lambda}{3H^2} \end{aligned}$$

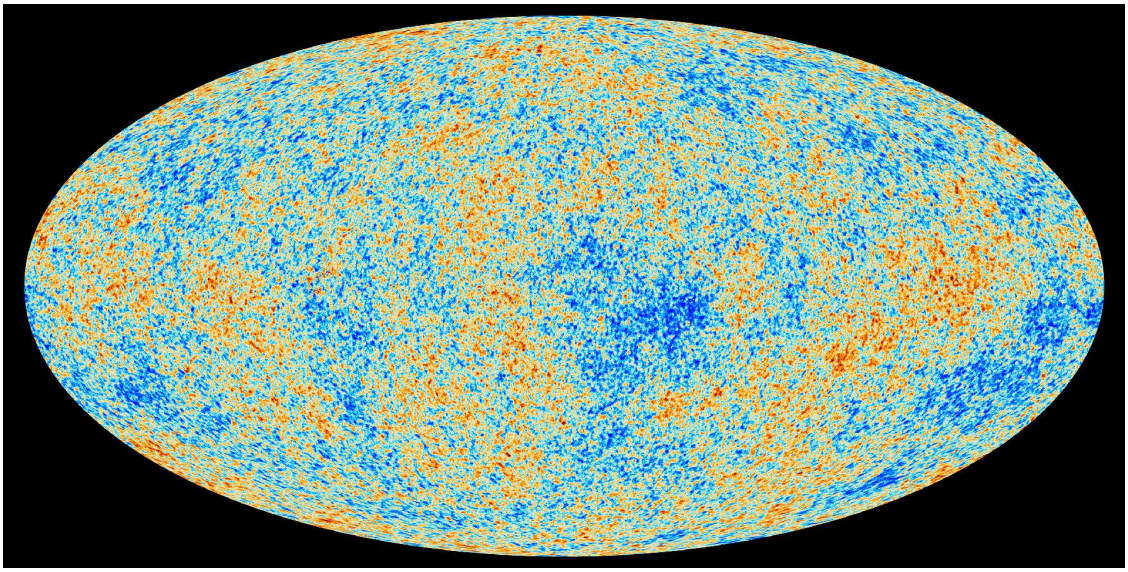
CMB measures Ω



COBE



WMAP



Planck

(iii) X-rays emitted from hot gases in elliptic galaxies

[M = total mass, L = luminosity]

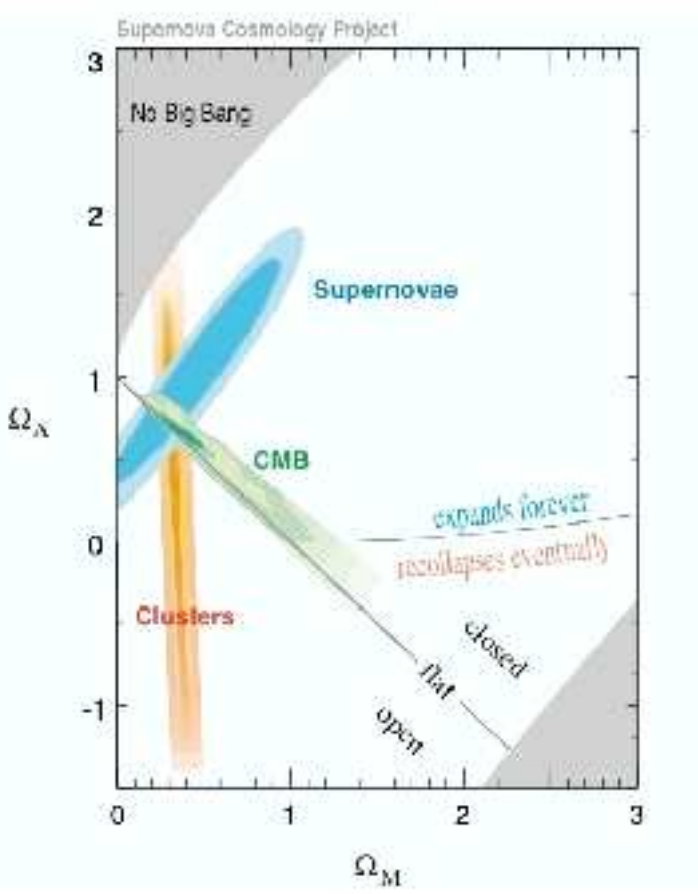
$$\Omega \approx 1 \Rightarrow \left(\frac{M}{L}\right)_c \sim 1400 \text{ [in solar units]}$$

measured: $\frac{M}{L} \lesssim 200 \ldots 500 \Rightarrow \Omega_m = \frac{M/L}{(M/L)_c} \lesssim 0.3\text{–}0.4$

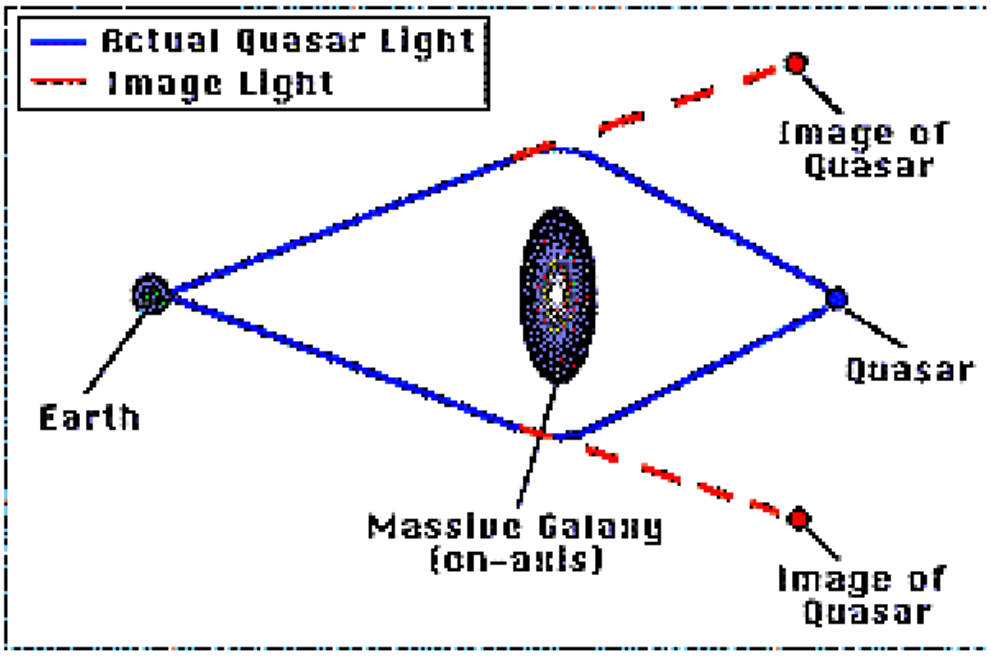
$\Rightarrow$ indication for DM [compatible with CMB]

Planck + WMAP + Supernovae + Cluster:

$$\begin{aligned} \Omega &= 1.005 \pm 0.006 \\ \Omega_\Lambda &= 0.692 \pm 0.010 \\ \Omega_B &= 0.0482 \pm 0.0005 \\ \Omega_{CDM} &= 0.258 \pm 0.004 \end{aligned}$$



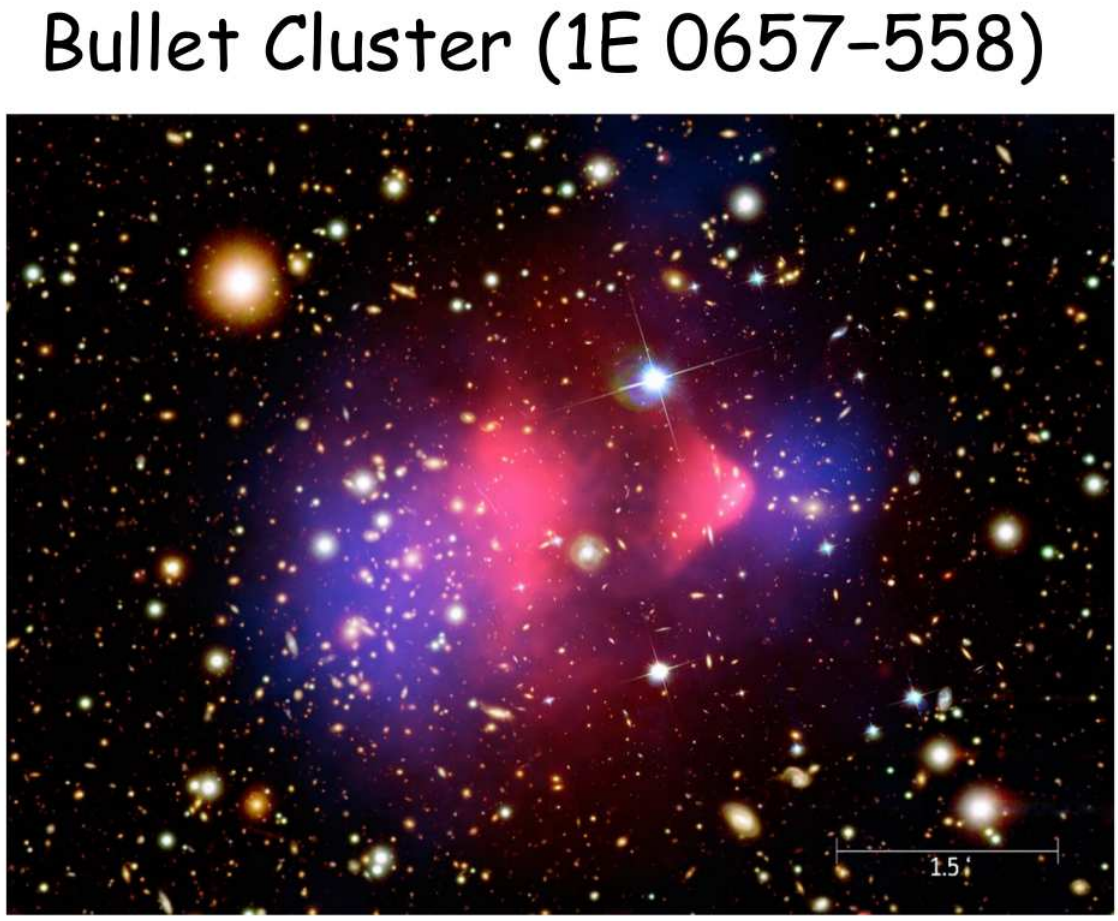
(iv) gravitational lenses:





$\Rightarrow$ Dark Matter

(v) bullet cluster:



Stars: telescope X-rays: pink Dark Matter: blue (rec. from grav. lensing)

$\Rightarrow$ Dark Matter

- Why $\Omega \approx 1$? [$\rightarrow k \approx 0$]

Big Bang theory: adiabatic expansion
$\Rightarrow R \sim T_\gamma^{-1}$

$$\Rightarrow \hat{k} = \frac{k}{R^2 T_\gamma^2} < \frac{8\pi G_N \rho}{3T_{CMB}^2} < 2 \cdot 10^{-58} \qquad \text{constant}$$

[$T_{CMB} = 2.725$ K]
$\rightarrow$ curvature unchanged [$\leftarrow$ contradiction]

solution: inflation theory

before inflation: $R = R_i, \quad T = T_i, \quad R \sim T^{-1}$

inflation: $R \sim e^{Ht} \quad [H = const]$
[$\leftarrow$ e.g. cosmological constant]

after inflation: $R = R_f \gg R_i, \quad T = T_f \lesssim T_i$

$\Rightarrow \hat{k} \rightarrow 0 \quad \Rightarrow \quad \boxed{\Omega \approx 1}$
[prediction]

• What is DM?

HDM: $m_\chi \lesssim 100$ eV
$\Rightarrow$ large structures develop first
galaxies much later
cannot reproduce present universe

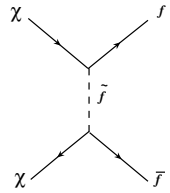
CDM: $m_\chi \gtrsim 1$ MeV
$\Rightarrow$ small structures form first
large structures later
$\Rightarrow$ compatible with today's universe

$\Rightarrow$ CDM

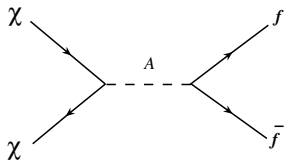
supersymmetry: R parity conserved
$\Rightarrow$ LSP stable [color and charge neutral]

candidates: $\tilde{\nu}$ excluded in MSSM [LEP]
$\tilde{G}$ not excluded
$\tilde{\chi}^0$ best candidate

relic density: Boltzmann equation for matter in expanding universe
$\leftarrow$ annihilation cross section $\sigma_{ann}(\tilde{\chi}\tilde{\chi} \to X)$

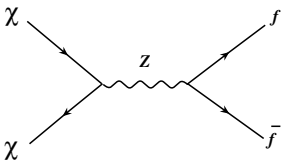


$A^{\tilde{f}}_{f\bar{f}} \propto (\frac{m_\chi}{m_{\tilde{f}}})^2$

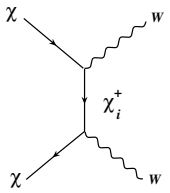


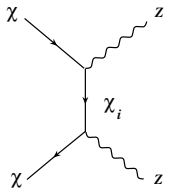
$A^{A}_{f\bar{f}} \propto \frac{\tan\beta m_{f_d}}{m_W}(\frac{m_\chi}{m_A})^2 N_1 N_{3(4)}$

$\tan\beta m_{f_d} \leftrightarrow \frac{m_{f_u}}{\tan\beta}$



$A^{Z}_{f\bar{f}} \propto (\frac{m_f m_\chi}{m_Z^2}) N^2_{3(4)}$





$A^{\chi_i/\chi_i^+}_{W^+W^-/ZZ} \propto \frac{1}{1+(m_{\chi_i^+}/m_\chi)^2-(m_W/m_\chi)^2}$

$(m_{\chi_i^+}, m_W) \leftrightarrow (m_{\chi_i}, m_Z)$

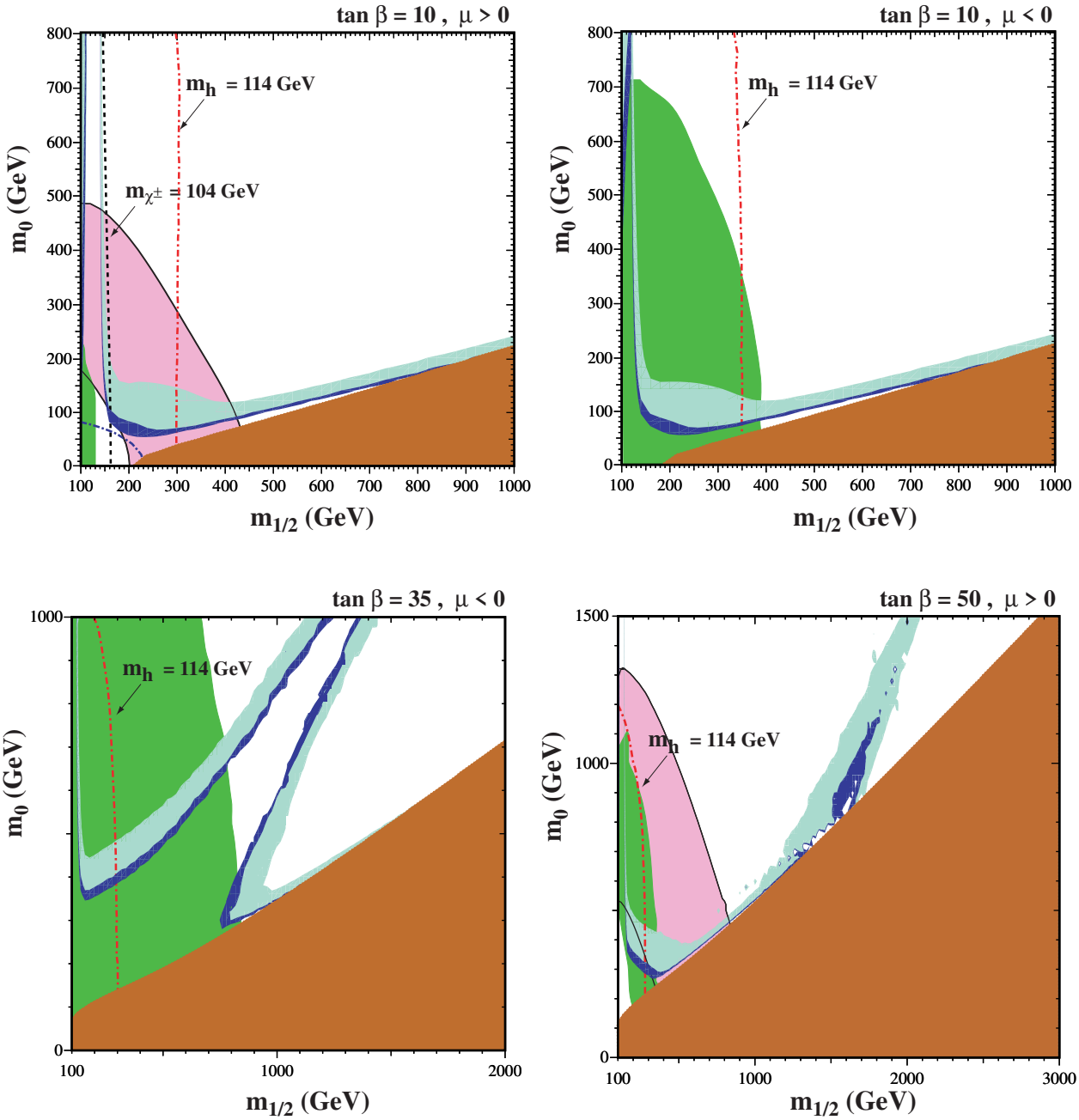
$$\rho_\chi = m_\chi n_\chi \text{ with } n_\chi \propto \frac{1}{\sigma_{ann}(\tilde{\chi}\tilde{\chi} \to X)} \propto m_\chi^2$$

$\Rightarrow$ relic density grows with increasing neutralino mass m_χ

during formation of galaxies:

before WMAP: $0.1 \lesssim \Omega_\chi H^2 \lesssim 0.3$

after WMAP: $0.094 \lesssim \Omega_\chi H^2 \lesssim 0.129$

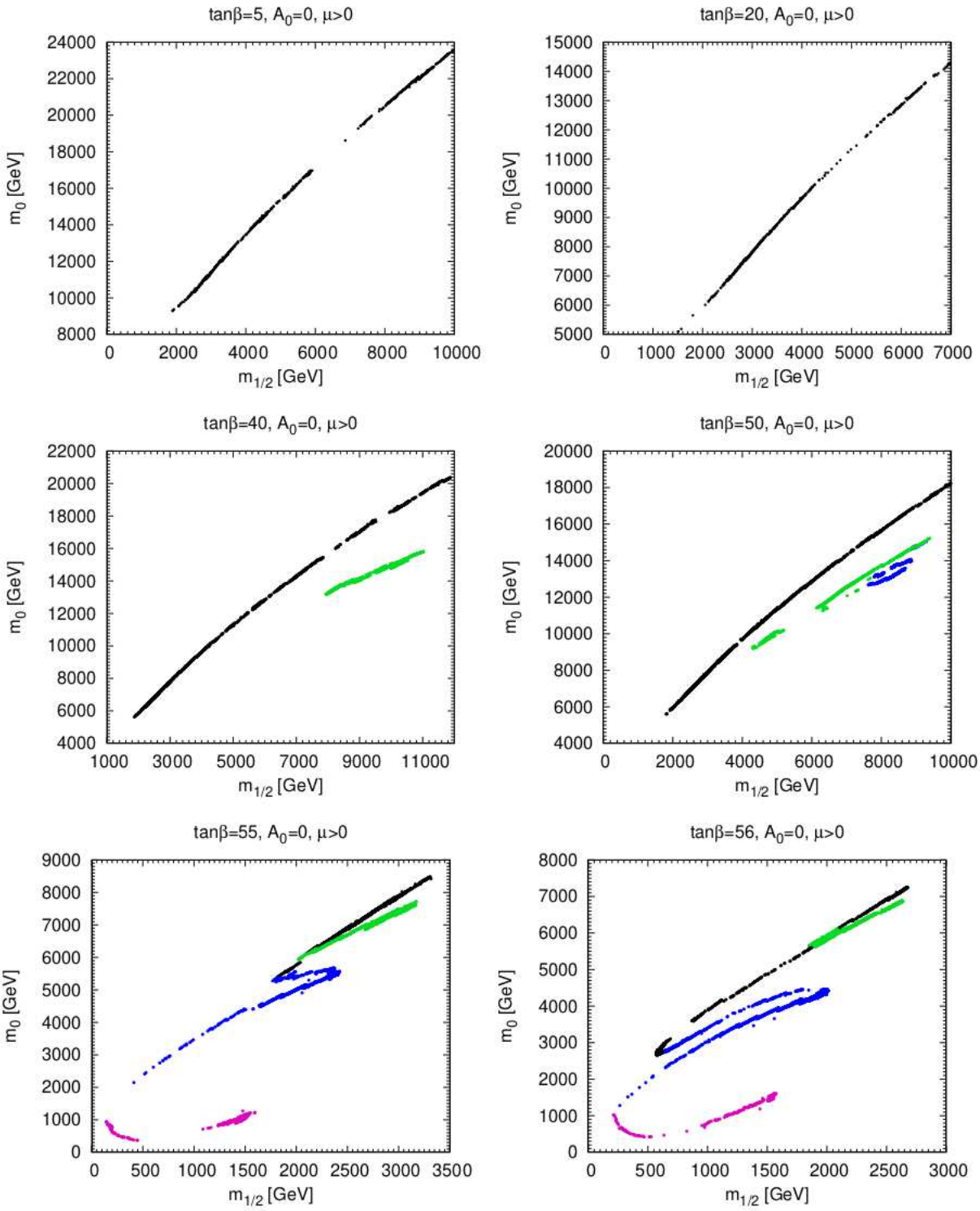


small $m_0, m_{1/2}$: bulk region

large $m_{1/2}$: coannihilation tail

large m_0: focus point [LSP=higgsino]

large tgβ: funnel regions [$2m_\chi \sim M_A \Rightarrow$ resonant s-channel]



Ellis, Olive, Spanos, Stamou

after Planck, LHC, LUX-ZEPLIN, Fermi-LAT, H.E.S.S., Icecube

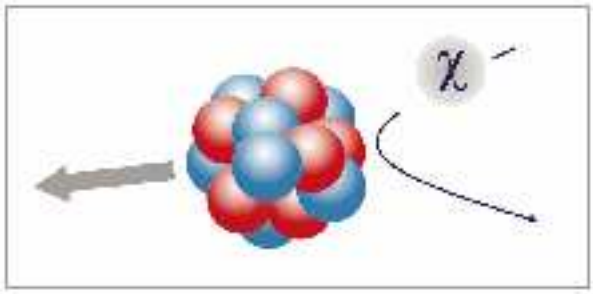
black: focus point [LSP=higgsino]

green: LSP=bino with higgsino component ['well-tempered']

purple: funnel region

blue: funnel region + coannihilation ['well-tempered']

- direct detection: elastic scattering $\chi\mathcal{N} \to \chi\mathcal{N}$



effective Lagrange density:

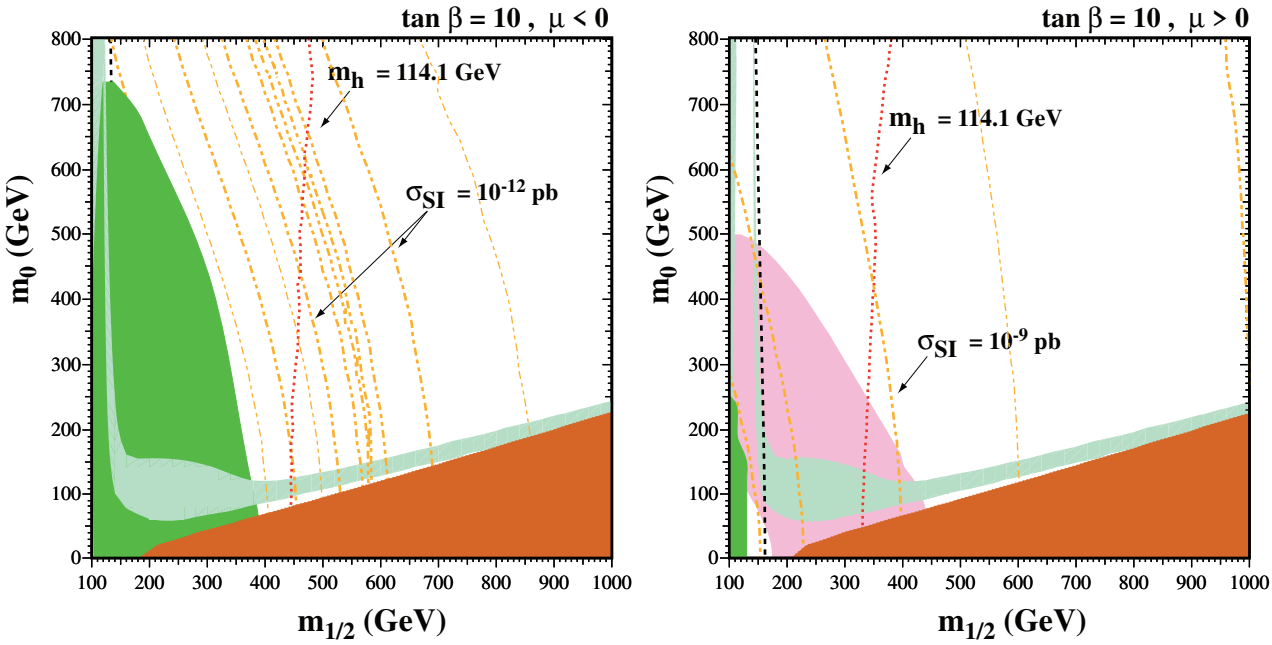
$$\begin{aligned}\mathcal{L} &= \bar{\chi}\gamma^{\mu}\gamma^{5}\chi\bar{q}_i\gamma_{\mu}(\alpha_{1i} + \alpha_{2i}\gamma^{5})q_i \\ &+ \alpha_{3i}\bar{\chi}\chi\bar{q}_i q_i + \alpha_{4i}\bar{\chi}\gamma^{5}\chi\bar{q}_i\gamma^{5}q_i \\ &+ \alpha_{5i}\bar{\chi}\chi\bar{q}_i\gamma^{5}q_i + \alpha_{6i}\bar{\chi}\gamma^{5}\chi\bar{q}_i q_i\end{aligned}$$

$\alpha_{1,i}, \alpha_{4,i}, \alpha_{5,i}, \alpha_{6,i} \to$ velocity-dependent elastic cross section

$\alpha_{2,i} \to$ spin-dependent cross section [$\leftarrow$ dominated by light $\tilde{q}$-exchange]

$\alpha_{3,i} \to$ scalar coefficient

spin-independent cross sections constrain $m_0, m_{1/2}$:



constraints from $b \to s\gamma$ and $g_\mu - 2$

$$\Rightarrow 2 \times 10^{-10} \text{ pb} \lesssim \quad \sigma_{SI} \quad \lesssim 6 \times 10^{-8} \text{ pb}$$
$$2 \times 10^{-7} \text{ pb} \lesssim \quad \sigma_{SD} \quad \lesssim 10^{-5} \text{ pb}$$

for $\mu > 0$

global analysis: spin-independent elastic scattering seems to be first accessible

• indirect detection: χ's are gravitationally bound by massive objects
$\Rightarrow$ several annihilation processes possible:
$\chi\chi \to \gamma + X, \nu + X, e^+ + X$

(i) γ's from annihilation in galactic center:
$\chi\chi \to \gamma\gamma, Z\gamma$ loop-suppressed $\Rightarrow$ continuum-γ's

γ-flux:

$$\Phi_\gamma(E_{thr}) = 5.6 \times 10^{-10}\ \text{cm}^{-2}\ \text{s}^{-1}$$
$$\times \sum_i \int_{E_{thr}}^{m_\chi} dE \frac{dN_\gamma^i}{dE} \left(\frac{\sigma_i v}{\text{pb}}\right) \left(\frac{100\ \text{GeV}}{m_\chi}\right)^2 \bar{J}(\Delta\Omega)\, \Delta\Omega\ ,$$

sum over all annihilation channels
$\Delta\Omega$: solid angle that is covered by γ-telescope
$\bar{J}$: measure for lumpiness of density profile of the galactic halo [huge uncertainties: $\bar{J} \sim 3 \ldots 10^5$!]

(ii) neutrinos from annihilation in sun and earth:

$\Rightarrow$ ν's propagate and convert to μ's via charged current reactions

μ-rates depend on primary annihilation process: $\chi\chi \to b\bar{b}, t\bar{t}, W^+W^-, ZZ$

(iii) e^+'s from annihilation in galactic halo:

positron flux:

$$\frac{d\Phi_{e^+}}{d\Omega dE} = \frac{\rho^2}{m_\chi^2} \sum_i \sigma_i v B^i_{e^+} \int dE_0 \, f_i(E_0) \, G(E_0, E) \ ,$$

sum over all annihilation channels

ρ: local neutralino mass density

$B^i_{e^+}$: branching fraction of channel i into e^+

$f(E_0)$: initial positron energy distribution

$G(E_0, E)$: Green-function for propagation of positrons in the galaxy