§8. MSSM Higgs Bosons

Higgs Potential:

$$\begin{aligned} V &= (m_1^2+\mu^2)|H_1|^2+(m_2^2+\mu^2)|H_2|^2 \\ &- B\mu\epsilon_{ij}\left(H_1^i H_2^j + h.c.\right) \\ &+ \frac{g^2+g'^2}{8}\left[|H_1|^2-|H_2|^2\right]^2+\frac{g^2}{2}|H_1^\dagger H_2|^2 \\ H_1 &= \begin{pmatrix} H_1^1 \\ \\ H_1^2 \end{pmatrix} = \begin{pmatrix} \phi_1^{0*} \\ \\ -\phi_1^- \end{pmatrix} \\ H_2 &= \begin{pmatrix} H_2^1 \\ \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ \\ \phi_2^0 \end{pmatrix} \end{aligned}$$

$\leftarrow$ quartic couplings $\sim$ gauge couplings

$\leftarrow$ diagonalisation:

$$\begin{aligned} H_1^1 &= \frac{1}{\sqrt{2}}\left(v_1 + H^0 c_\alpha - h^0 s_\alpha + iA^0 s_\beta - iG^0 c_\beta\right) \\ H_1^2 &= H^- s_\beta - G^- c_\beta \\ H_2^1 &= H^+ c_\beta + G^+ s_\beta \\ H_2^2 &= \frac{1}{\sqrt{2}}\left(v_2 + H^0 s_\alpha + h^0 c_\alpha + iA^0 c_\beta + iG^0 s_\beta\right) \end{aligned}$$

$G^\pm, G^0 \Rightarrow$ Goldstone bosons

$\leftarrow$ parameters:

$$
\begin{aligned}
M_Z^2 &= \frac{g^2+g'^2}{4}(v_1^2+v_2^2) \\
M_A^2 &= B\mu\frac{v_1^2+v_2^2}{v_1 v_2} \\
\mathrm{tg}\beta &= \frac{v_2}{v_1} \\
m_1^2+\mu^2 &= -\frac{M_Z^2}{2}+(M_A^2+M_Z^2)\sin^2\beta \\
m_2^2+\mu^2 &= -\frac{M_Z^2}{2}+(M_A^2+M_Z^2)\cos^2\beta
\end{aligned}
$$

$\Rightarrow$ Higgs sector determined by 2 parameters $(M_A, \mathrm{tg}\beta)$

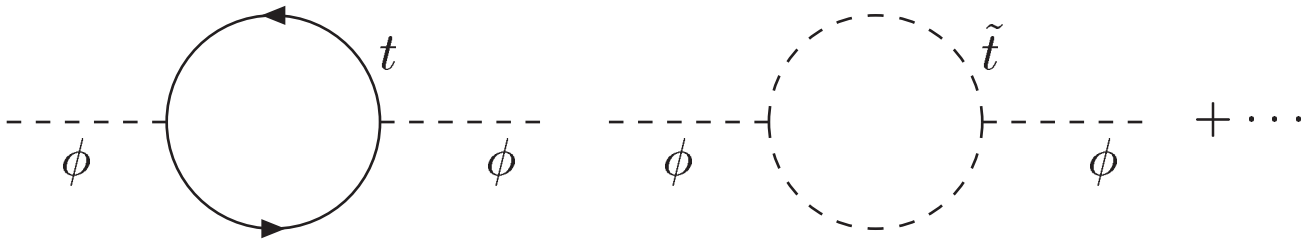
bilinear terms in V:

$$
\begin{aligned}
\mathcal{M}_\pm^2 &= \frac{1}{2}(M_A^2+M_W^2)s_{2\beta}\begin{pmatrix}\mathrm{tg}\beta & 1\\ 1 & \cot\beta\end{pmatrix} \\
&\Rightarrow M_{H^\pm}^2 = M_A^2+M_W^2 \qquad \text{charged} \\
\mathcal{M}_a^2 &= \frac{1}{2}M_A^2 s_{2\beta}\begin{pmatrix}\mathrm{tg}\beta & 1\\ 1 & \cot\beta\end{pmatrix} \Rightarrow M_A^2 \qquad \mathcal{CP}_o \\
\mathcal{M}_s^2 &= \frac{1}{2}s_{2\beta}\begin{pmatrix}M_A^2\mathrm{tg}\beta+M_Z^2\cot\beta & -M_A^2-M_Z^2\\ -M_A^2-M_Z^2 & M_A^2\cot\beta+M_Z^2\mathrm{tg}\beta\end{pmatrix} \\
\Rightarrow M_{h,H}^2 &= \frac{1}{2}\left\{M_A^2+M_Z^2\mp\sqrt{(M_A^2+M_Z^2)^2-4M_A^2M_Z^2c_{2\beta}^2}\right\} \qquad \mathcal{CP}_e \\
\mathrm{tg}2\alpha &= \mathrm{tg}2\beta\,\frac{M_A^2+M_Z^2}{M_A^2-M_Z^2} \qquad -\frac{\pi}{2}<\alpha<0
\end{aligned}
$$

Relations:

$$\begin{aligned} M_h &< M_Z, M_A \\ M_H &> M_Z, M_A \\ M_{H^\pm} &> M_A, M_W \end{aligned}$$

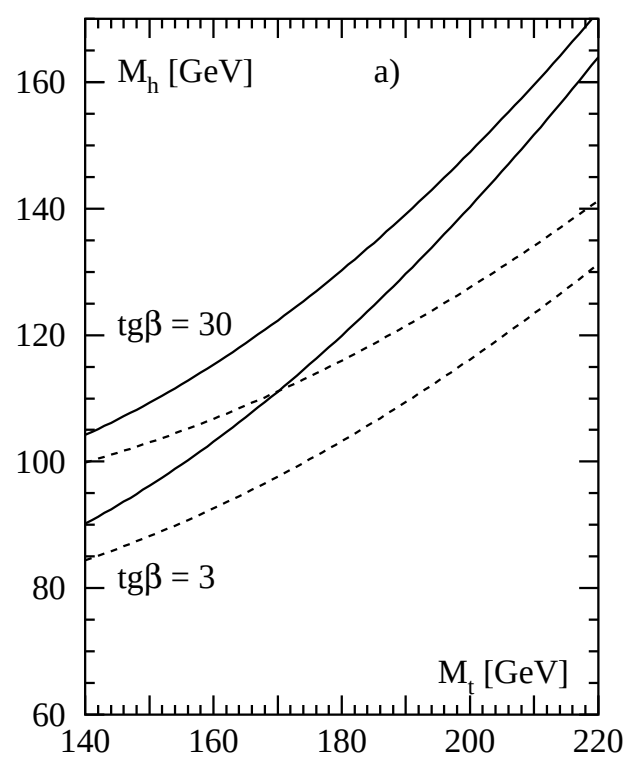
modified by radiative corrections

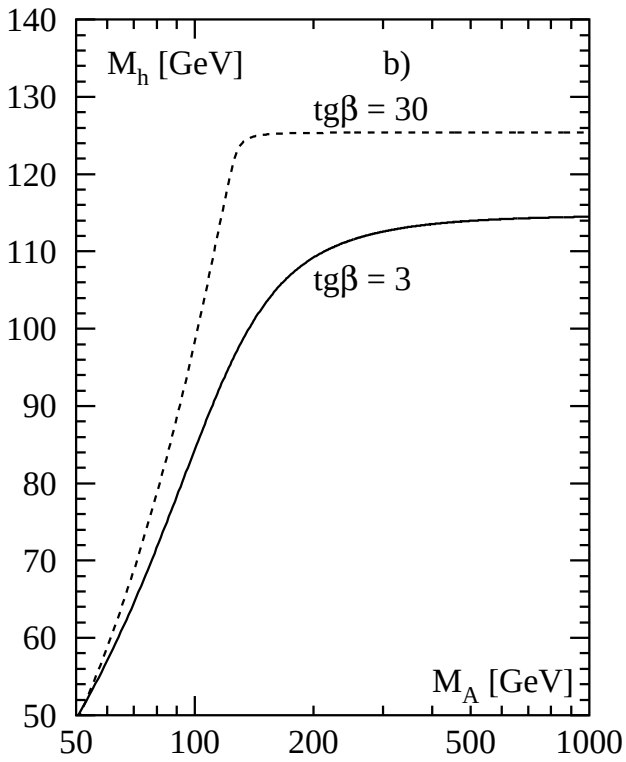


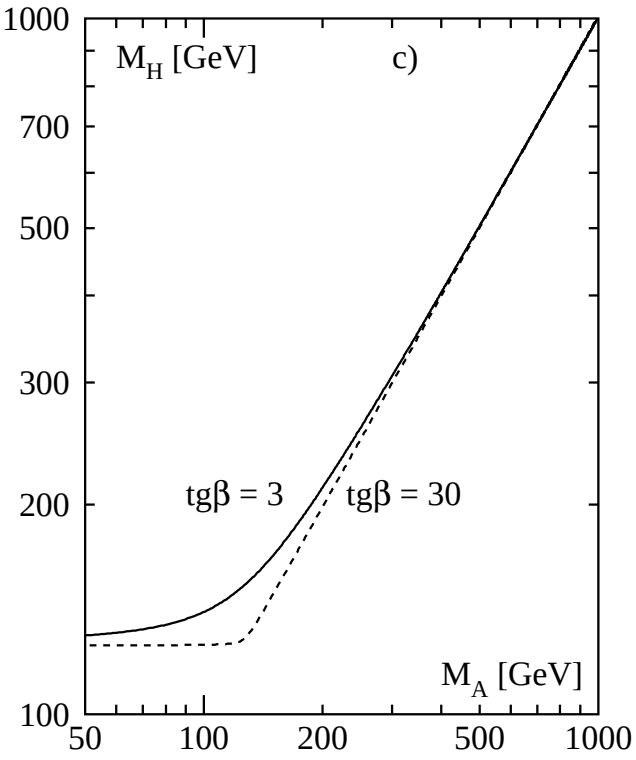
$$\epsilon = \frac{3G_F m_t^4}{\sqrt{2}\pi^2 s_\beta^2} \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \; + \; \text{stop mixing } [\mu, A_t]$$

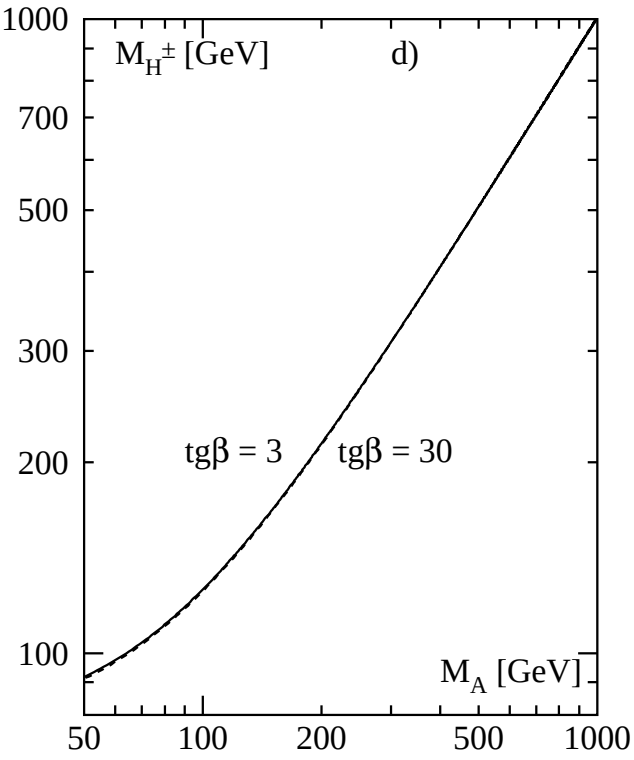
$\longrightarrow$ $M_h \lesssim 140$ GeV

$M_{H,A,H^\pm} = \ldots v \ldots 1$ TeV


M_h [GeV]
a)
tgβ = 30
tgβ = 3
M_t [GeV]
160
140
120
100
80
60
140
160
180
200
220


M_h [GeV]
b)
tgβ = 30
tgβ = 3
M_A [GeV]
140
130
120
110
100
90
80
70
60
50
50
100
200
500
1000


M_H [GeV]
c)
tgβ = 3
tgβ = 30
M_A [GeV]
1000
700
500
300
200
100
50
100
200
500
1000


M_H± [GeV]
d)
tgβ = 3
tgβ = 30
M_A [GeV]
1000
700
500
300
200
100
50
100
200
500
1000

Higgs couplings:

$$\begin{aligned}
\mathcal{L}_{int} &= \left|\left(ig\frac{\sigma^a}{2}W_\mu^a + ig'\frac{Y}{2}B_\mu\right)H_1\right|^2 \\
&+ \left|\left(ig\frac{\sigma^a}{2}W_\mu^a + ig'\frac{Y}{2}B_\mu\right)H_2\right|^2 \\
&- \lambda_L\overline{e_R}H_1^{c\dagger}L - \lambda_U\overline{u_R}H_2^{c\dagger}Q - \lambda_D\overline{d_R}H_1^{c\dagger}Q + h.c.
\end{aligned}$$

$$\Rightarrow g_\pm = -\frac{1}{\sqrt{2}}\left[\frac{m_u}{v}\cot\beta(1\mp\gamma_5) + \frac{m_d}{v}\mathrm{tg}\beta(1\pm\gamma_5)\right]$$

$$g_{\phi Q\bar{Q}} = g_Q^\phi\frac{m_Q}{v}$$

$$g_{\phi VV} = g_V^\phi\frac{M_V^2}{v}$$

ϕ	g_u^ϕ	g_d^ϕ	g_V^ϕ
H_{SM}	1	1	1
h	$\frac{\cos\alpha}{\sin\beta}$	$-\frac{\sin\alpha}{\cos\beta}$	$\sin(\beta-\alpha)$
H	$\frac{\sin\alpha}{\sin\beta}$	$\frac{\cos\alpha}{\cos\beta}$	$\cos(\beta-\alpha)$
A	$\cot\beta$	$\mathrm{tg}\beta$	0

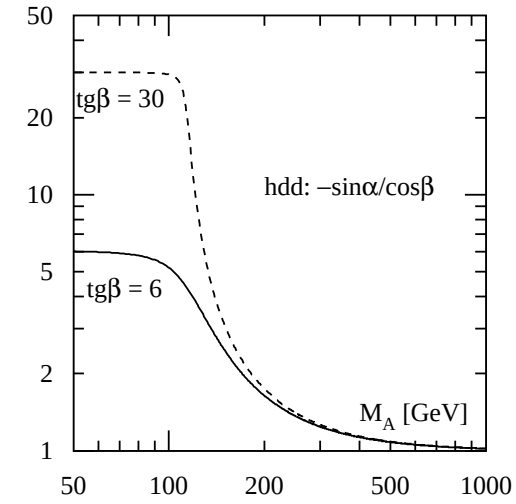
- $M_A^2 \gg M_Z^2$: $M_H \sim M_A \sim M_{H^\pm} \sim$ degenerate

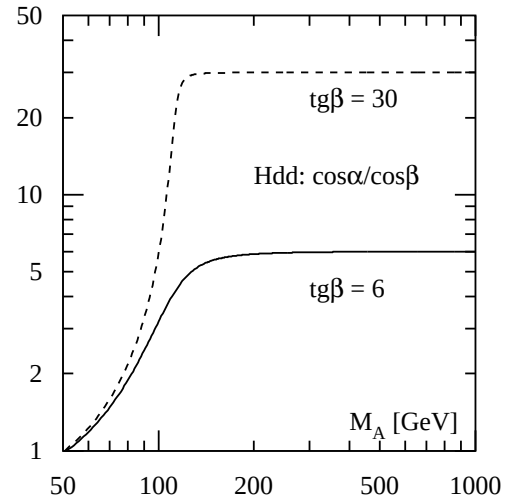
$$M_h \sim \sqrt{M_Z^2 \cos^2 2\beta + \epsilon \sin^2 \beta}$$

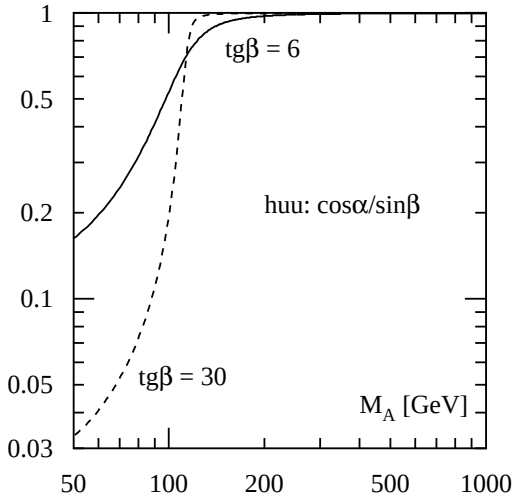
$$\alpha \sim \beta - \frac{\pi}{2}$$

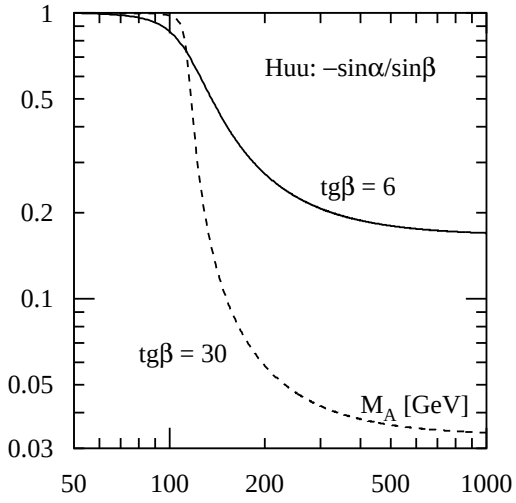
$$\Rightarrow h \sim H_{SM} \text{ [decoupling limit]}$$

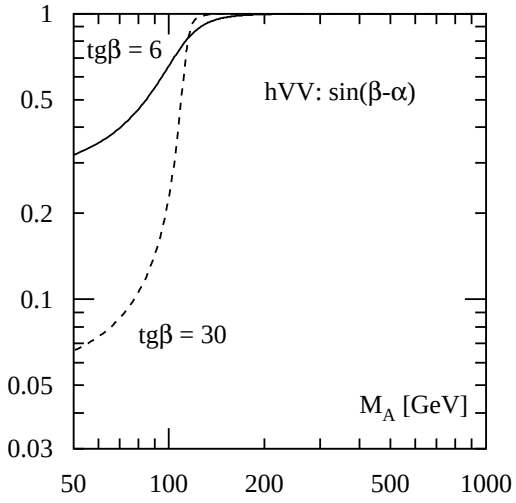
- tg$\beta\uparrow$ $\Longrightarrow$ $g_u^\phi\downarrow$ $g_d^\phi\uparrow$ $g_V^\phi\downarrow$

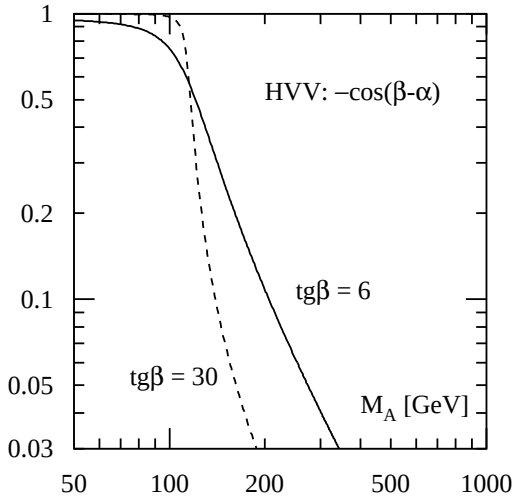

tgβ = 30
hdd: –sinα/cosβ
tgβ = 6
M_A [GeV]
50
20
10
5
2
1
50
100
200
500
1000


tgβ = 30
Hdd: cosα/cosβ
tgβ = 6
M_A [GeV]
50
20
10
5
2
1
50
100
200
500
1000


tgβ = 6
huu: cosα/sinβ
tgβ = 30
M_A [GeV]
1
0.5
0.2
0.1
0.05
0.03
50
100
200
500
1000

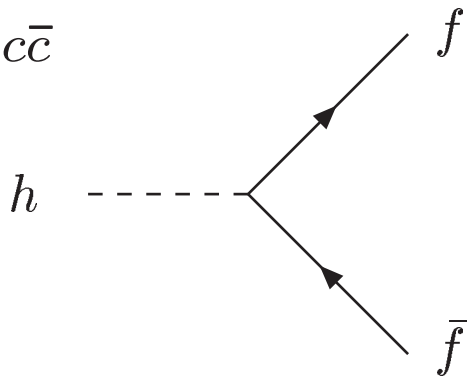

Huu: –sinα/sinβ
tgβ = 6
tgβ = 30
M_A [GeV]
1
0.5
0.2
0.1
0.05
0.03
50
100
200
500
1000


tgβ = 6
hVV: sin(β-α)
tgβ = 30
M_A [GeV]
1
0.5
0.2
0.1
0.05
0.03
50
100
200
500
1000

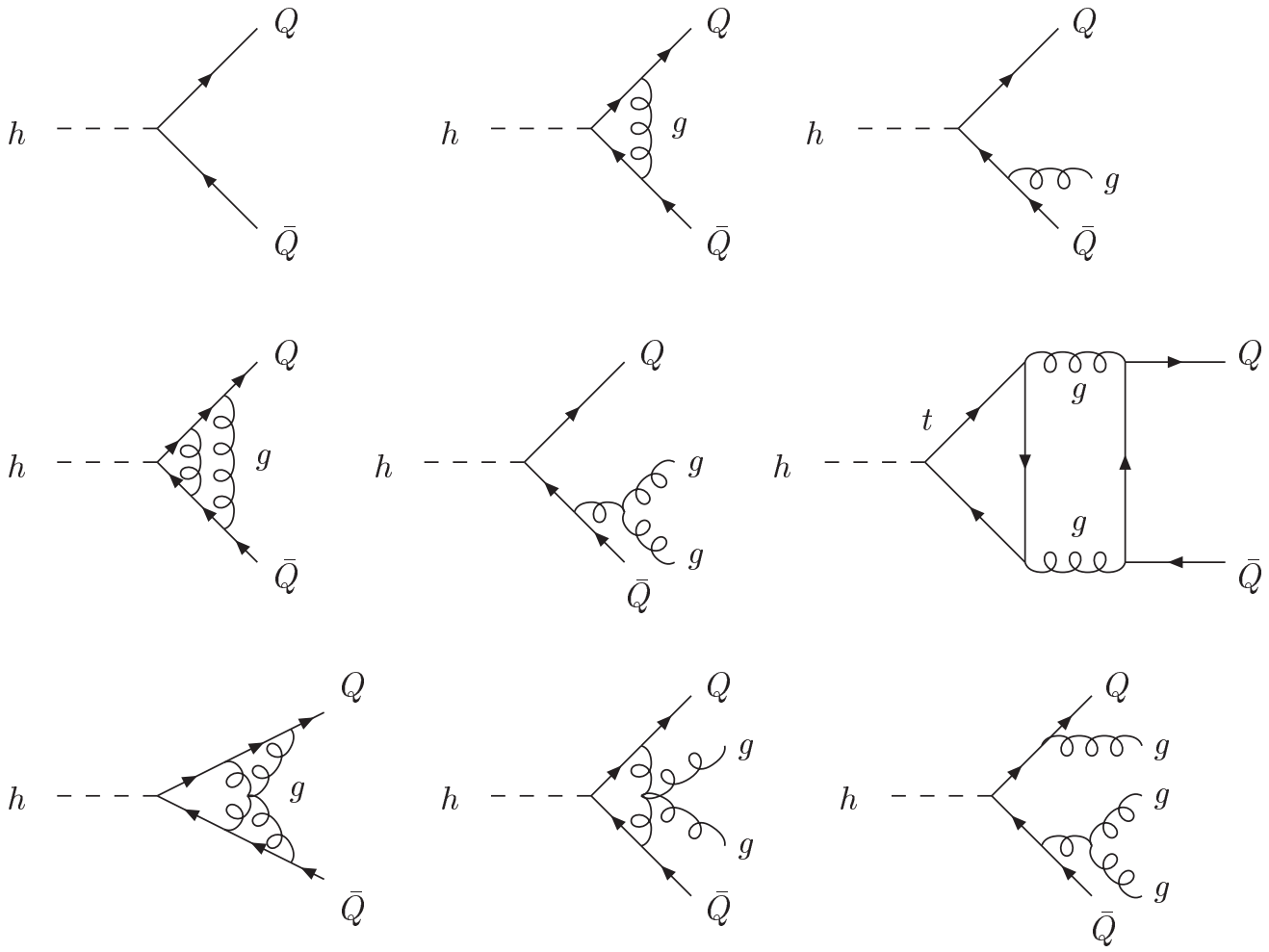

HVV: –cos(β-α)
tgβ = 6
tgβ = 30
M_A [GeV]
1
0.5
0.2
0.1
0.05
0.03
50
100
200
500
1000

Higgs Decays

- $h \to b\bar{b}, \tau^+\tau^-, c\bar{c}$



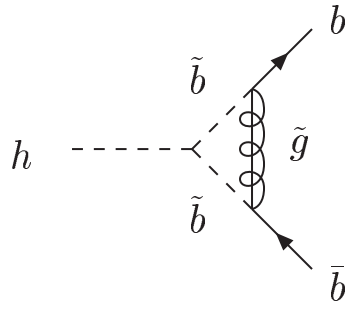
large QCD corrections: $\sim -50 \ldots -80\%$



dominant effect:

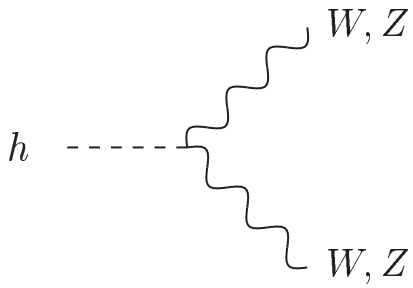
$$m_b \to \bar{m}_b(M_h) = m_b \left[1 - \frac{\alpha_s}{\pi}\left(\frac{4}{3} + \log\frac{M_h^2}{m_b^2}\right) + \mathcal{O}(\alpha_s^2)\right]$$

large SUSY–QCD corrections: $\mathcal{O}(10\ldots100\%)$



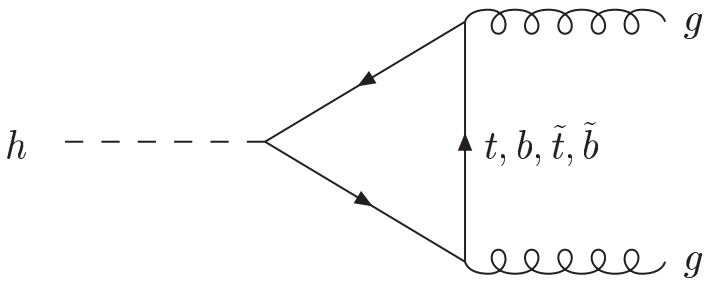
$+\cdots \qquad \propto \frac{\alpha_s}{\pi}\frac{m_{\tilde{g}}\mu \mathrm{tg}\beta}{m^2_{\tilde{b}}}$

- $h \to WW^*, ZZ^*$

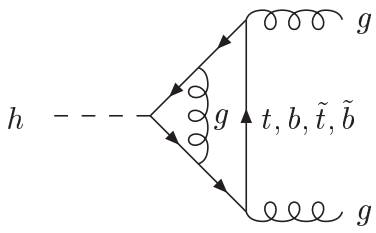


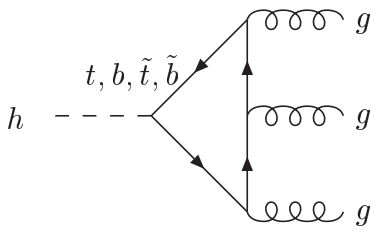
electroweak corrections: $\sim 5\ldots10\%$

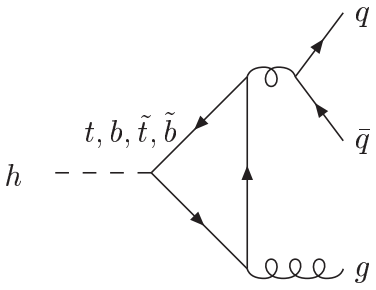
- $h \to gg$



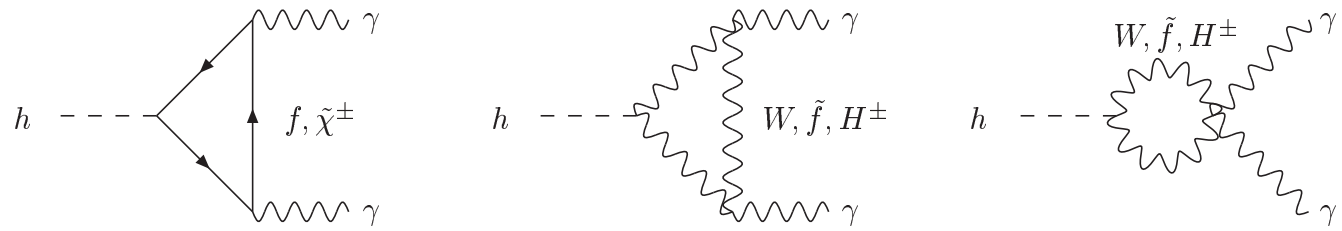
large QCD corrections: $\sim 10\ldots100\%$





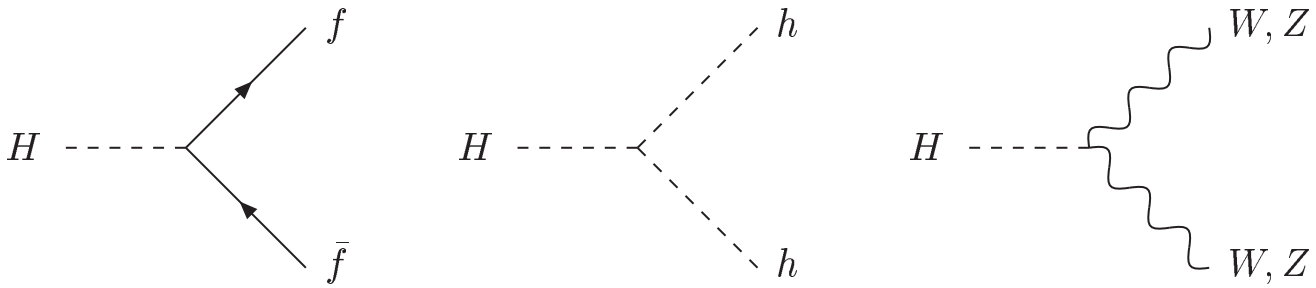


- $h \to \gamma\gamma$

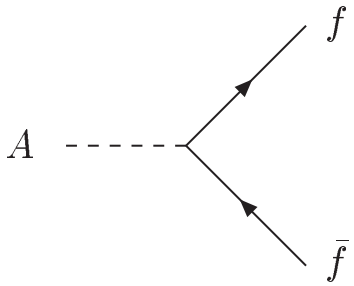


extremly important decay channel for LHC

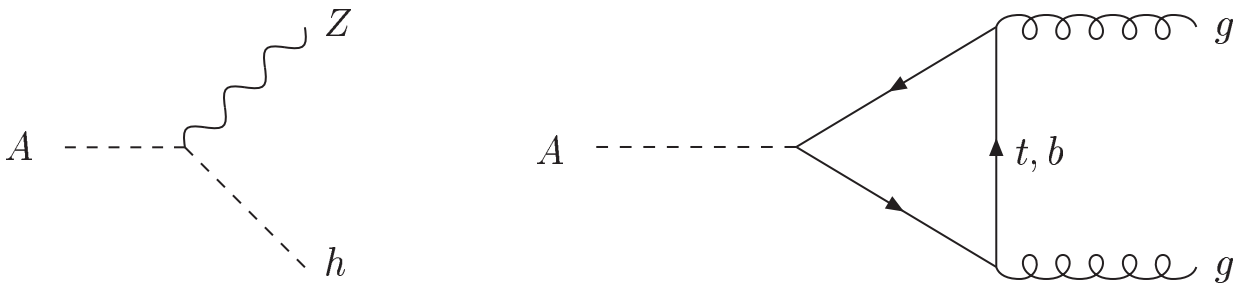
- $H \to b\bar{b}, \tau^+\tau^-$: dominant for large tg$\beta$
- $H \to hh, WW, ZZ, t\bar{t}$



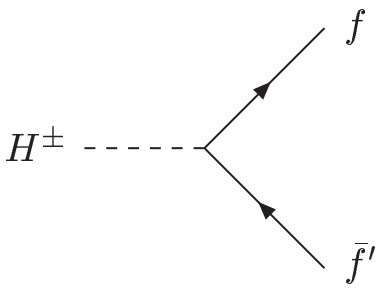
- $A \to b\bar{b}, \tau^+\tau^-$: dominant for large tg$\beta$
- $A \to t\bar{t}$: dominant above the $t\bar{t}$ threshold for small and moderate tgβ



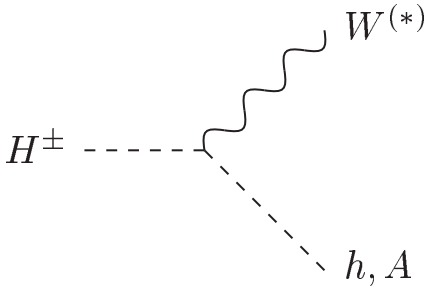
- $A \to Zh, gg$

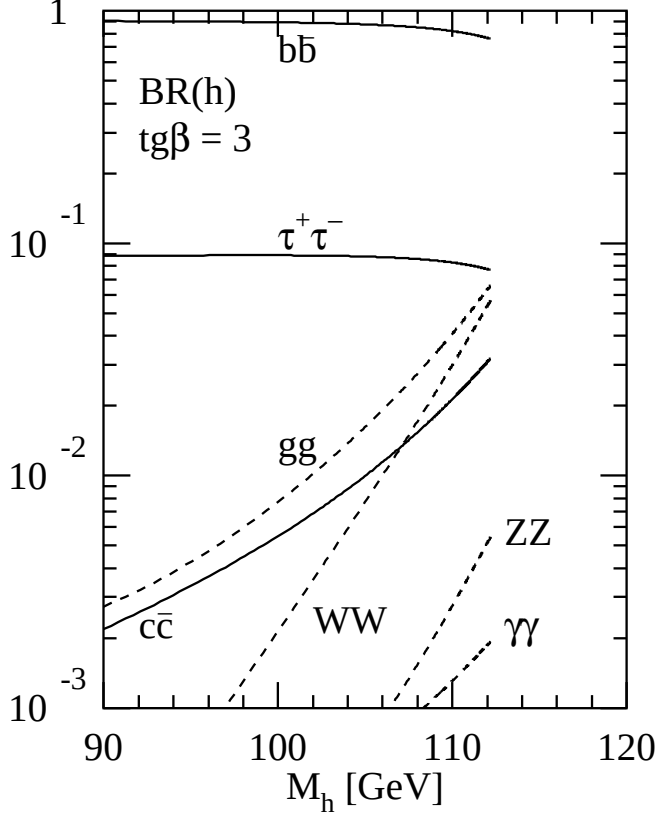


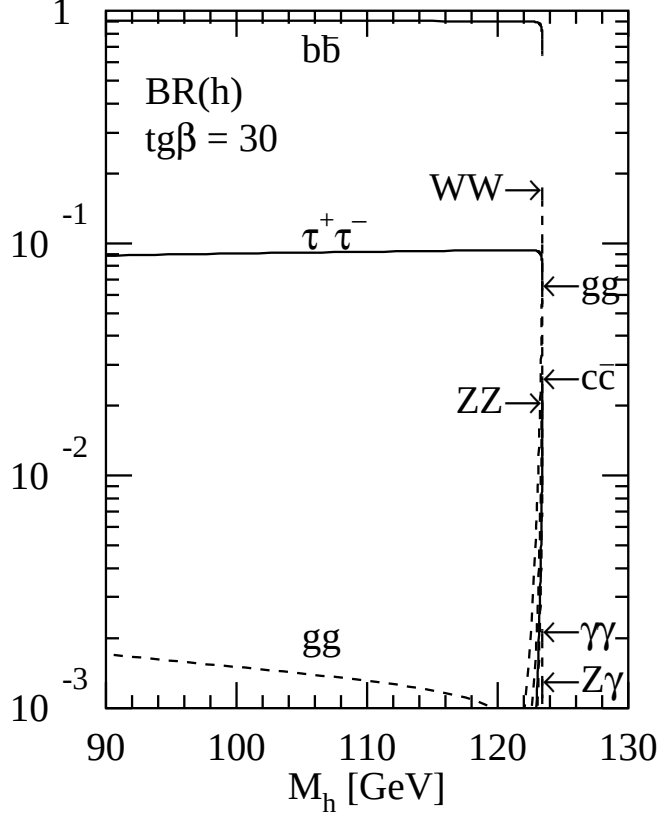
- $H^+ \to \tau^+\nu_\tau, t\bar{b}$

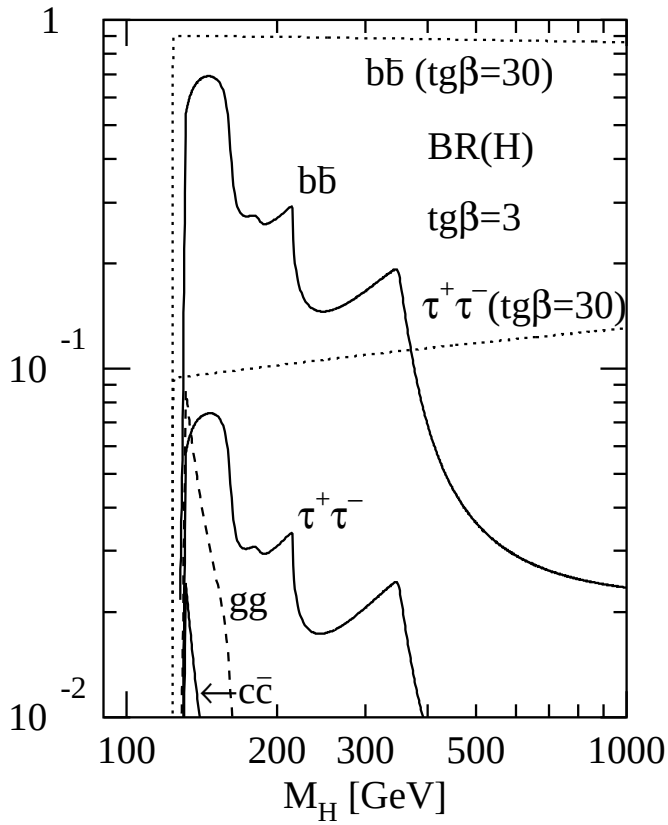


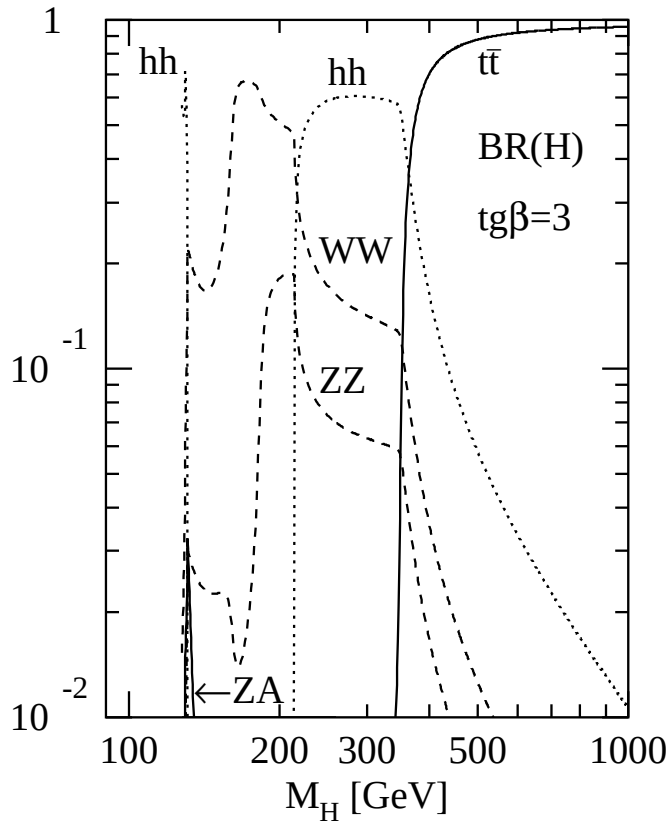
- $H^+ \to W^*A, Wh$



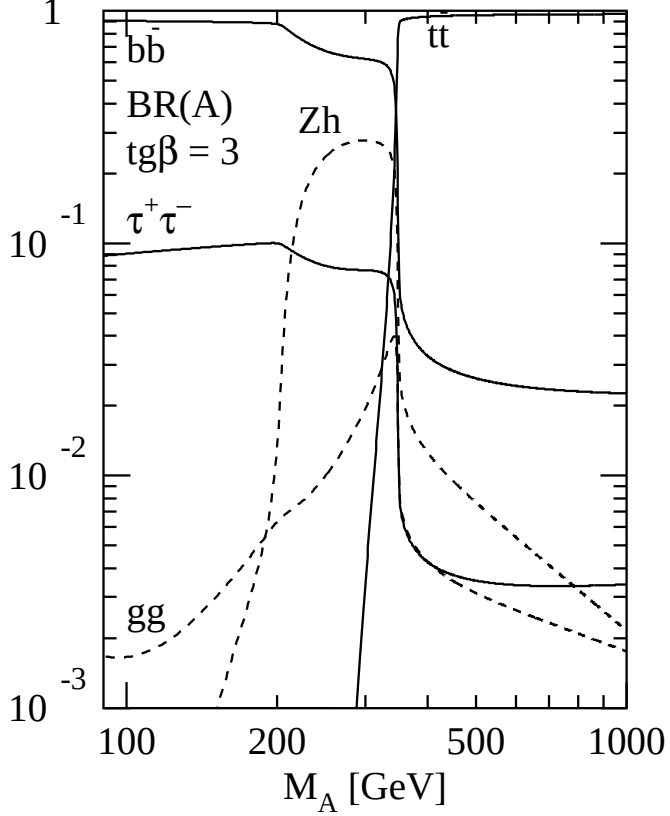


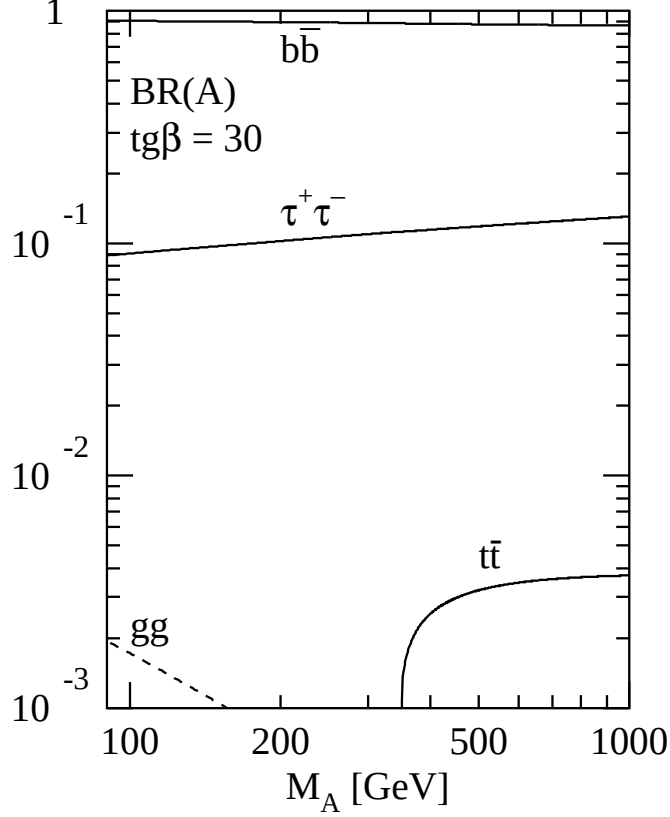


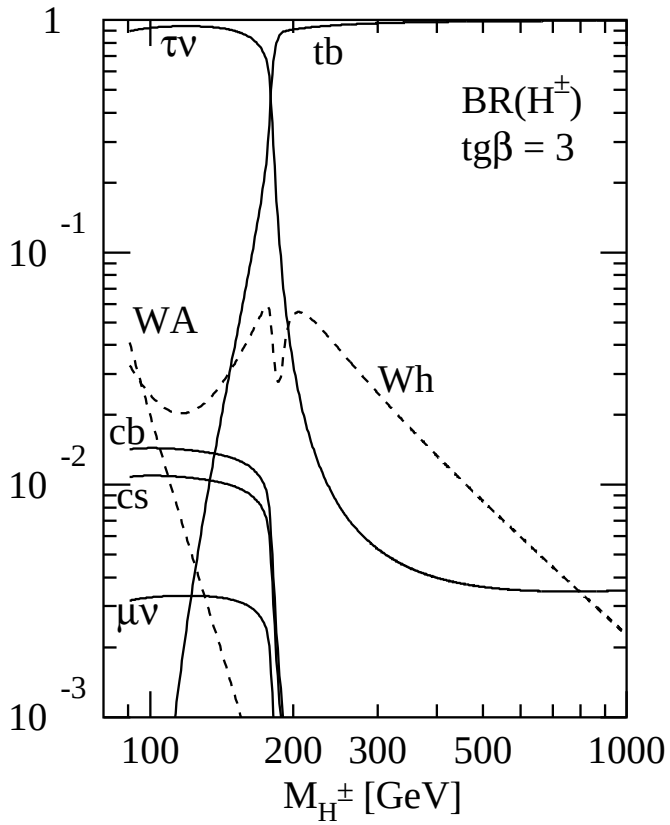


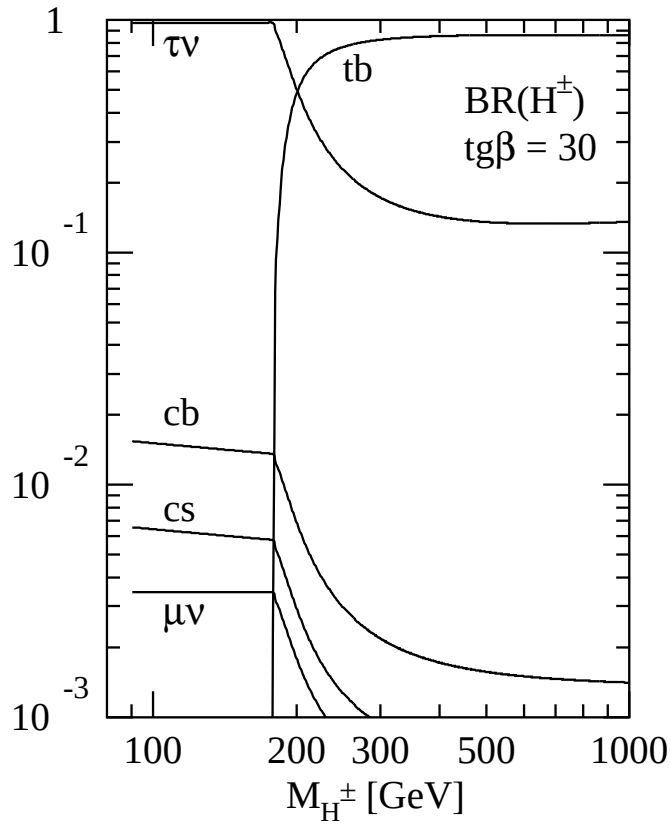


HDECAY [Djouadi, Kalinowski, S.]



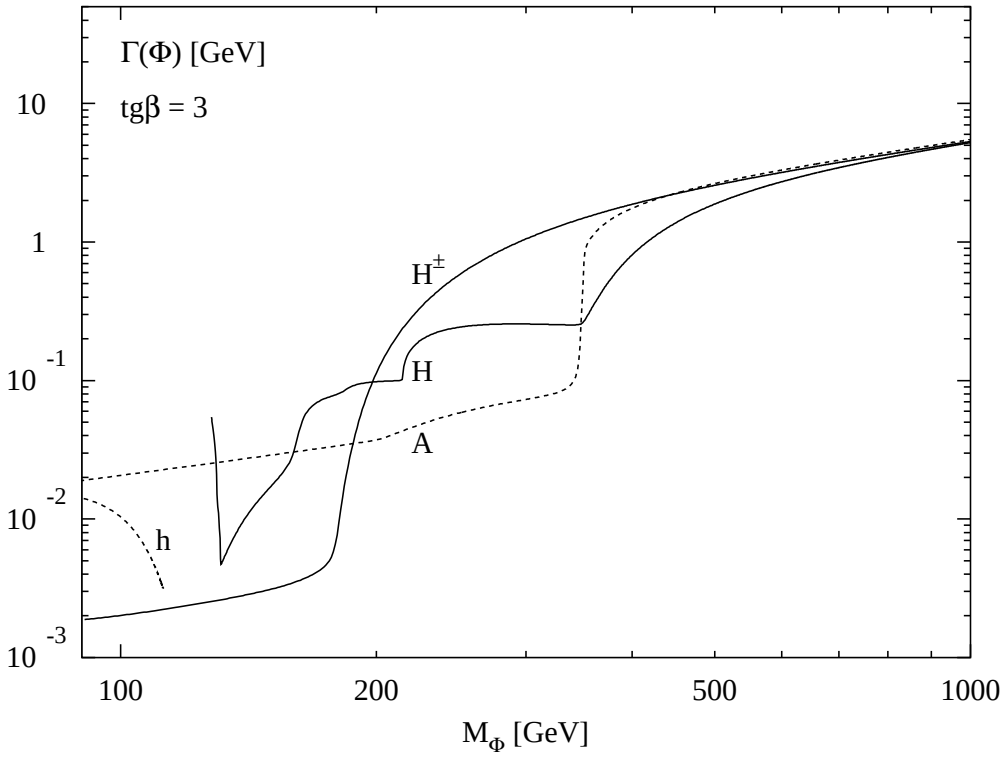


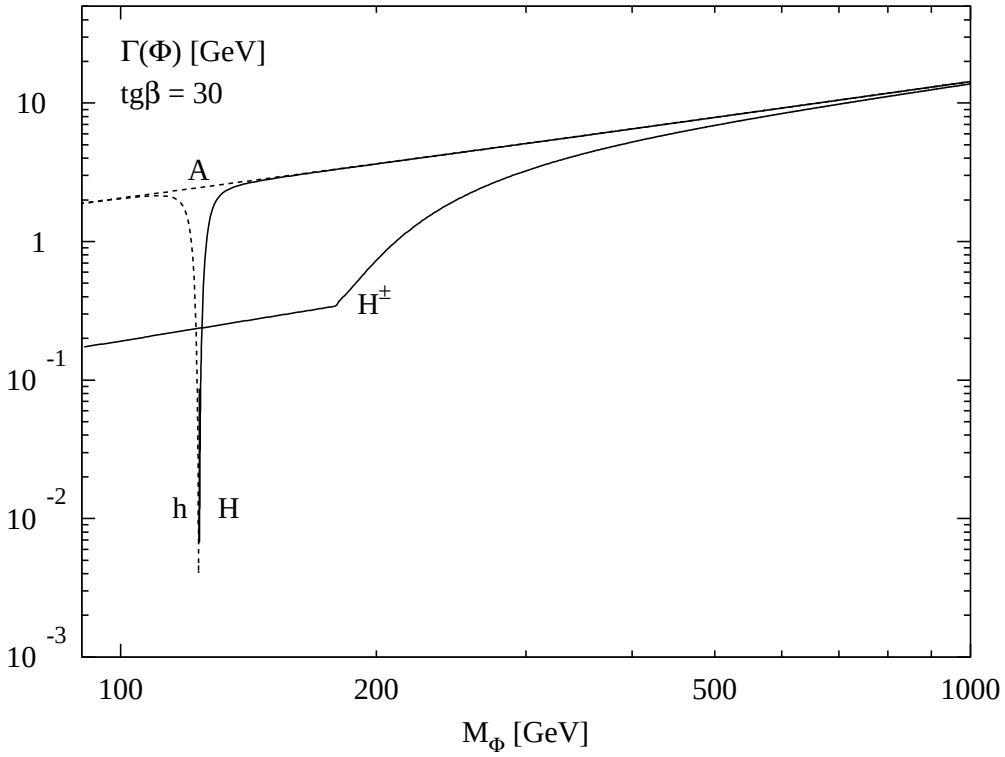




HDECAY [Djouadi, Kalinowski, S.]

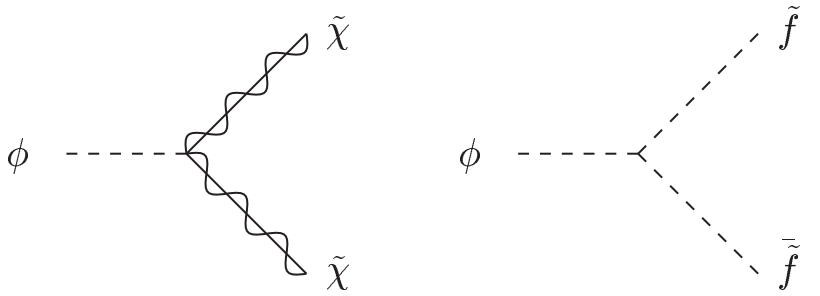
- total widths: $\Gamma_\phi \lesssim 10 - 30$ GeV narrow

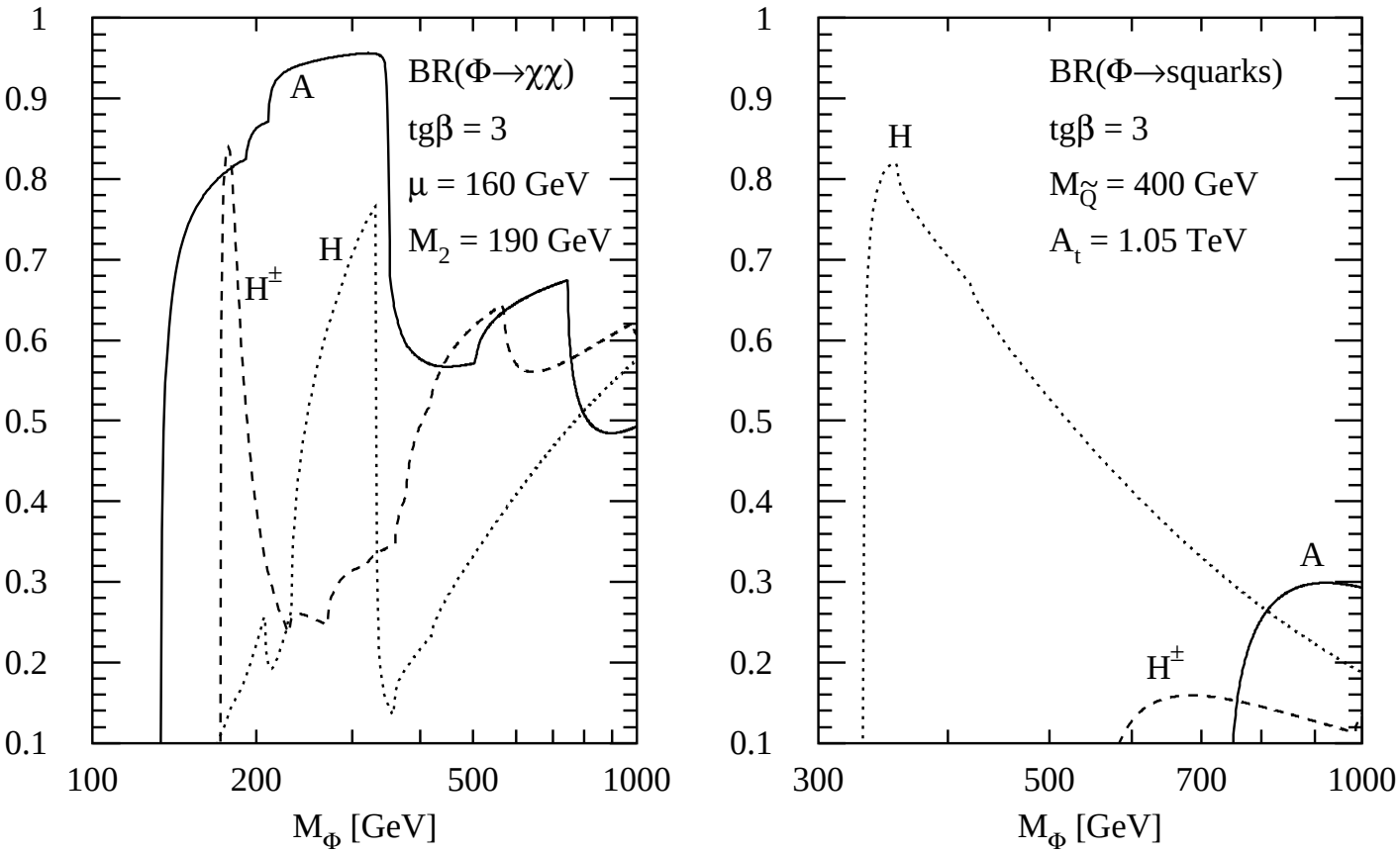




HDECAY [Djouadi, Kalinowski, S.]

• decays into gauginos and sfermions [3rd generation] important, if kinematically allowed



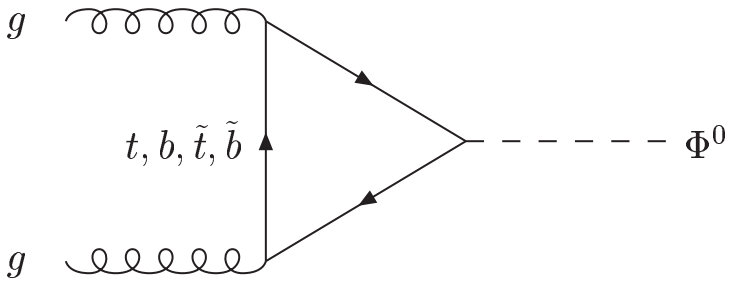


HDECAY [Djouadi, Kalinowski, S.]

Higgs Production

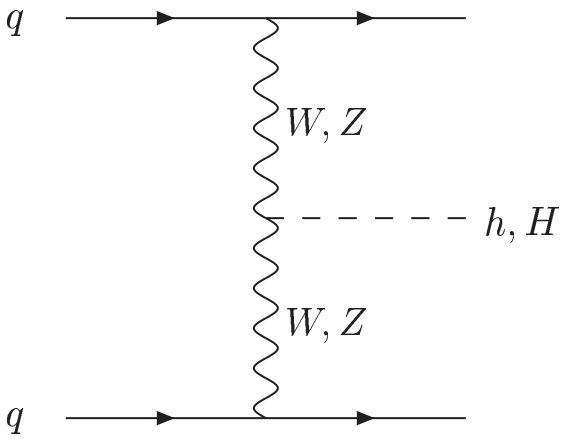
(a) Hadron colliders

- gluon fusion: $pp \to gg \to h/H/A$



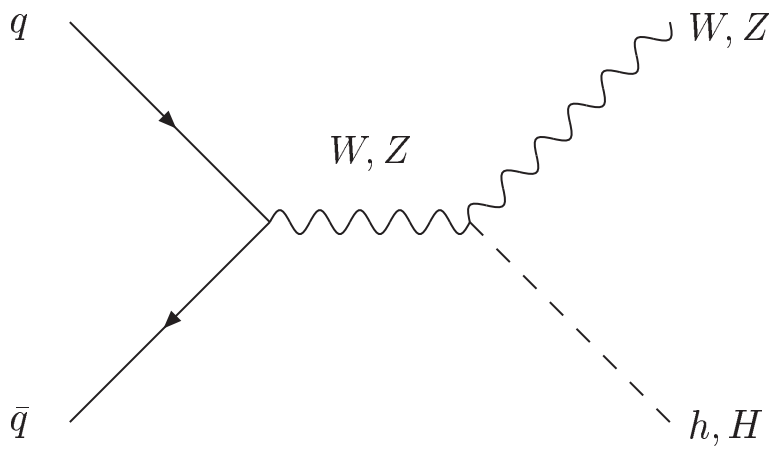
QCD corrections: $\sim 10 \ldots 100\%$

- W/Z fusion: $pp \to W^*W^*/Z^*Z^* \to h/H$



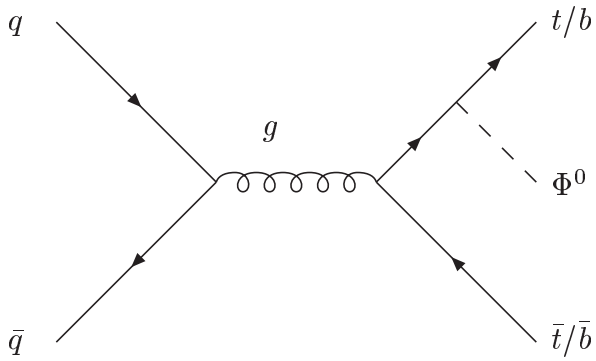
QCD corrections: $\sim 10\%$

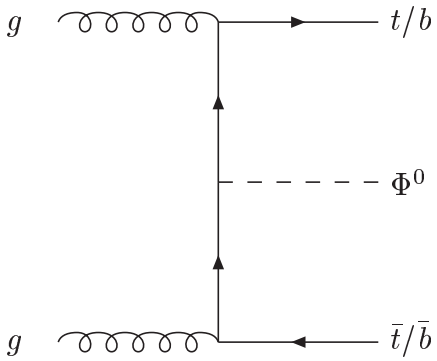
- Higgs–strahlung: $pp \to W^*/Z^* \to W/Z + h/H$



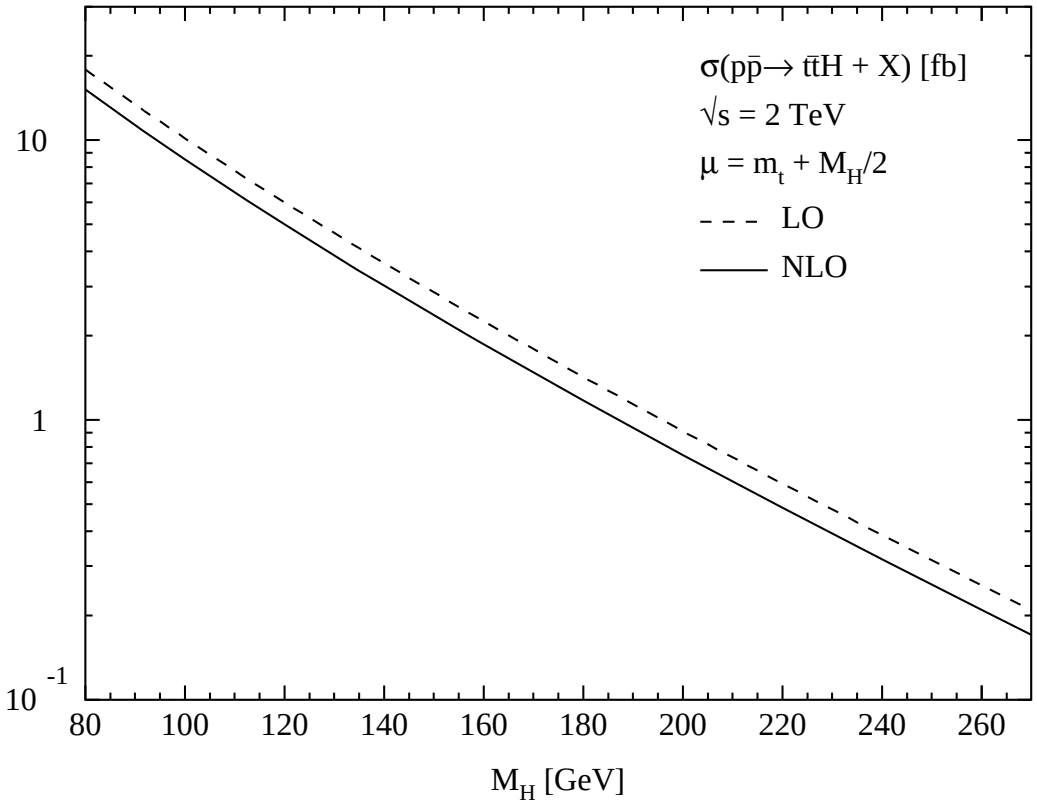
QCD corrections: $\sim 30\%$

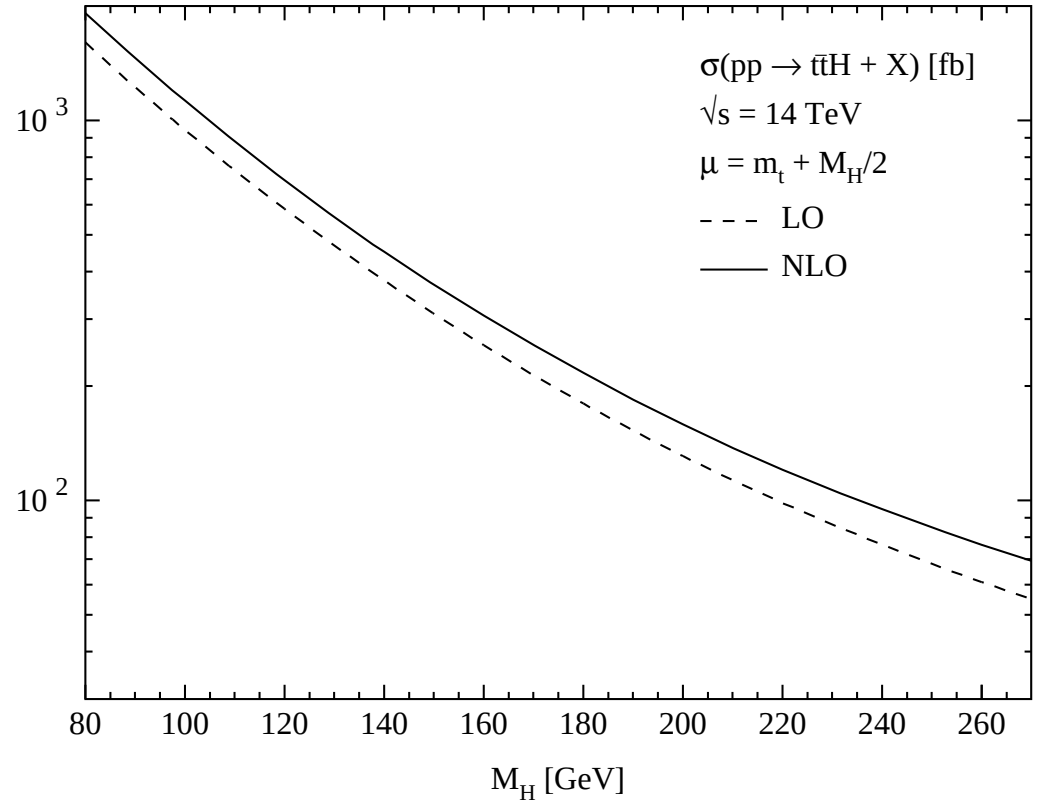
- bremsstrahlung: $pp \to t\bar{t}/b\bar{b} + h/H/A$



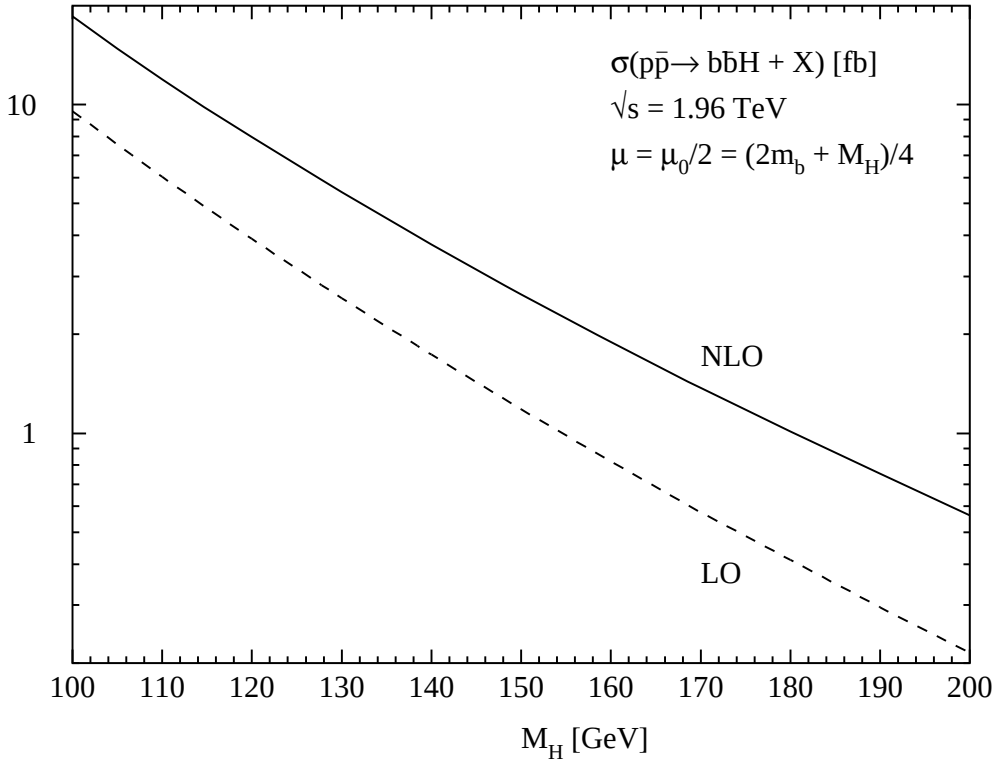


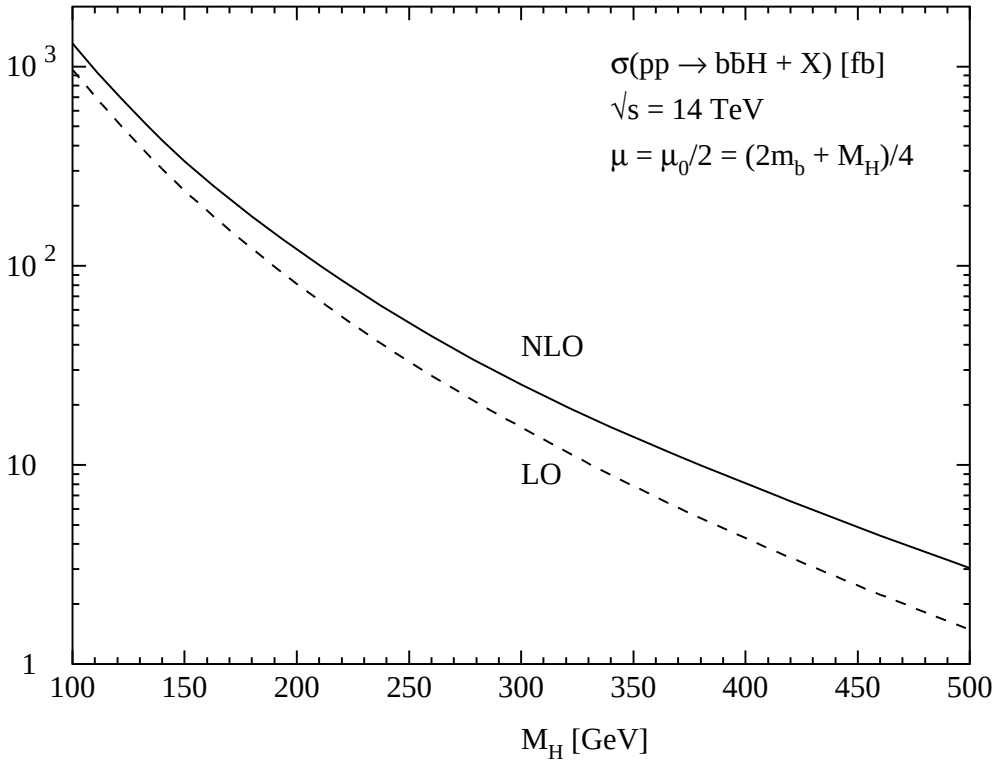
QCD corrections:
$t\bar{t}\phi^0$: $\sim \pm 20\%$
$b\bar{b}\phi^0$: $\sim +(50 - 100)\%$



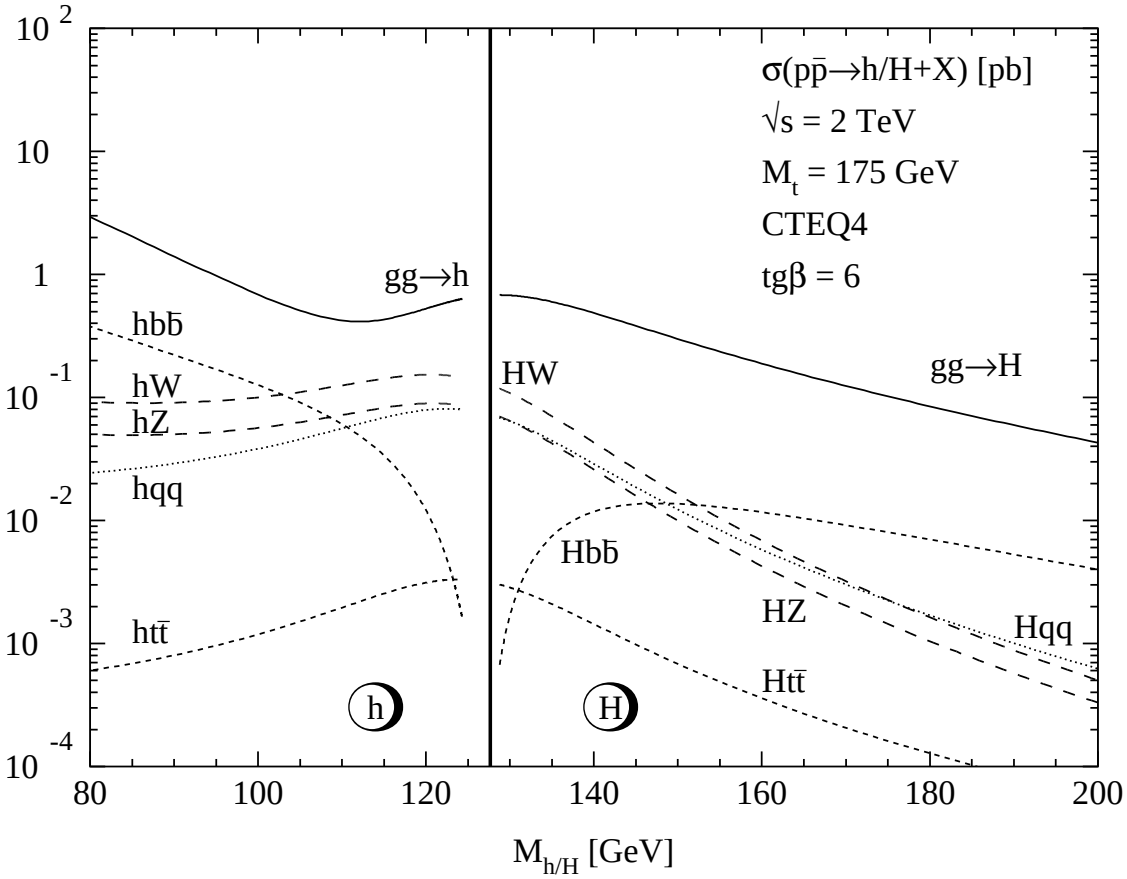


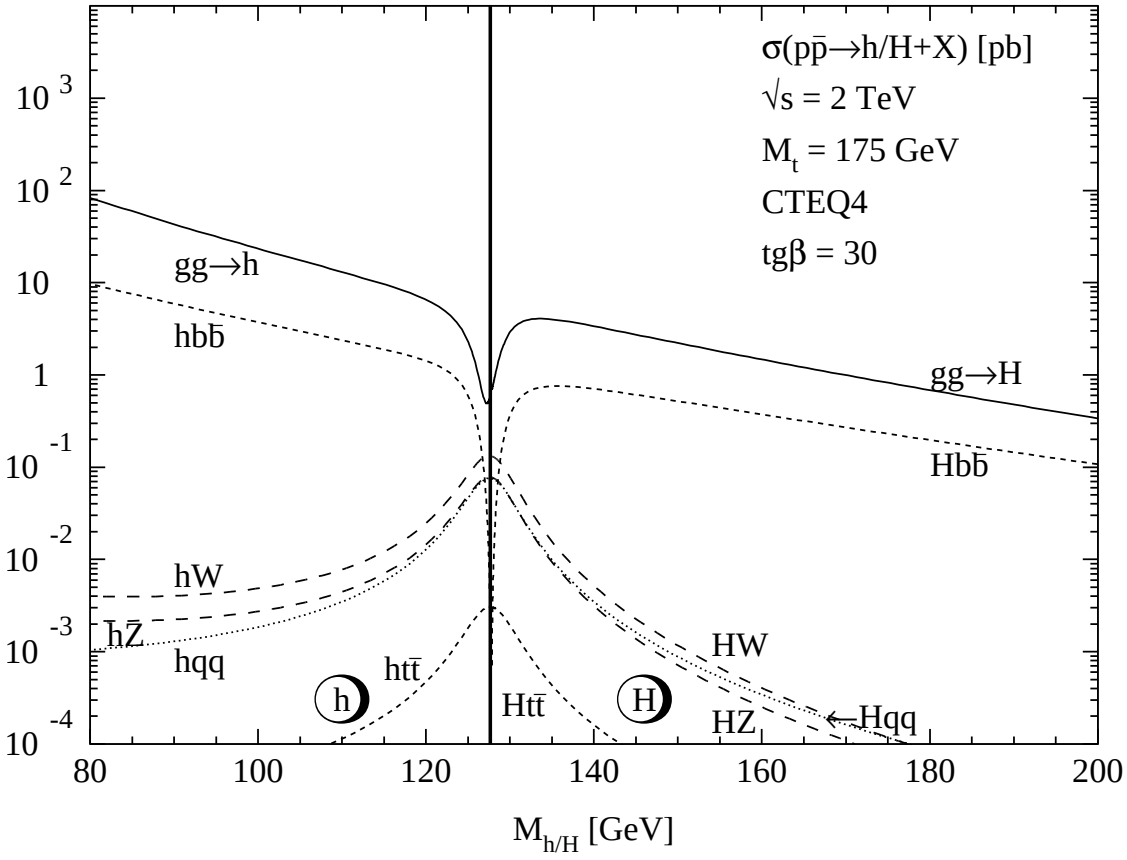
Beenakker, Dittmaier, Krämer, Plümper, S., Zerwas

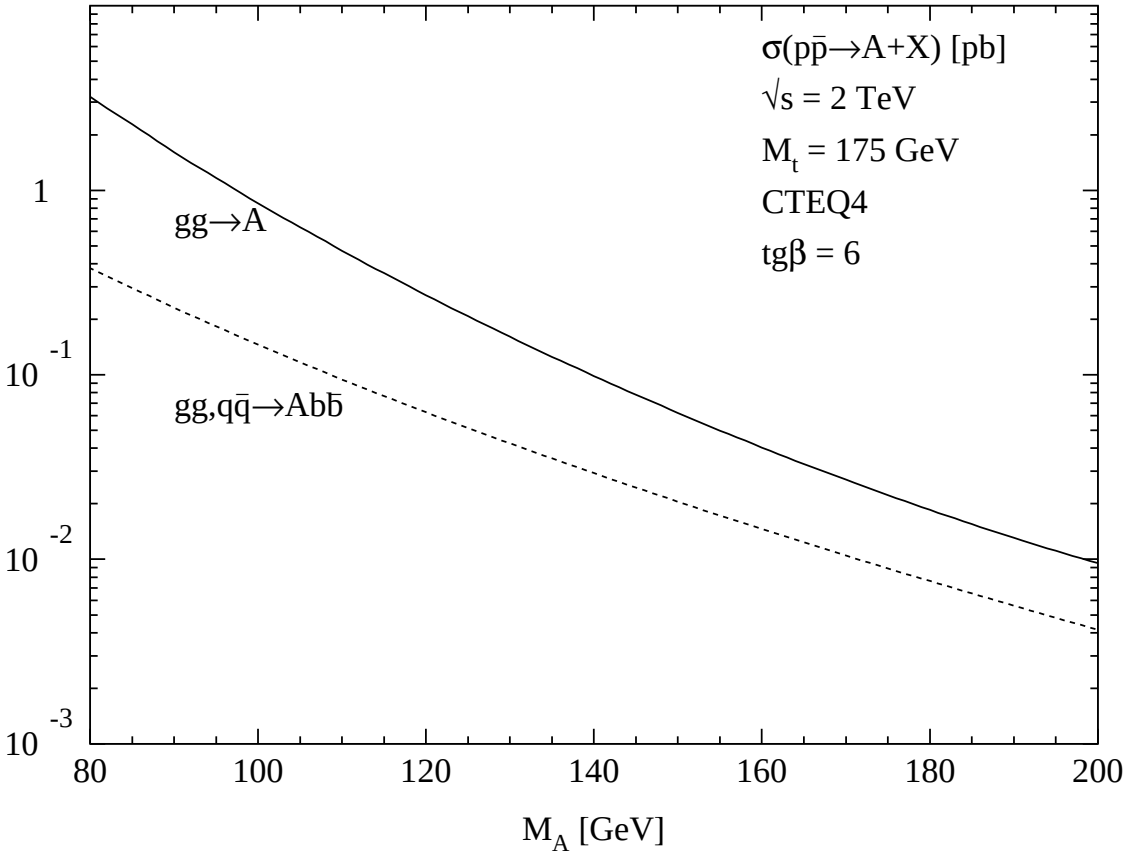


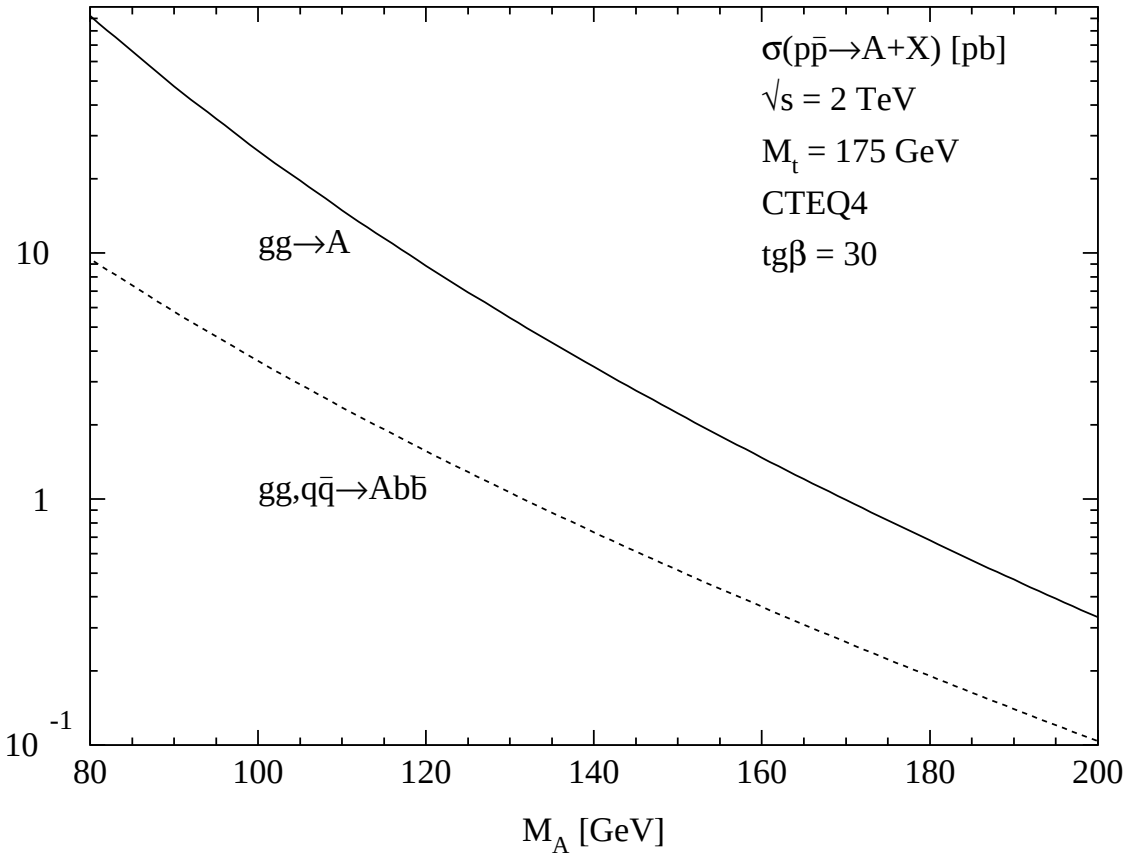


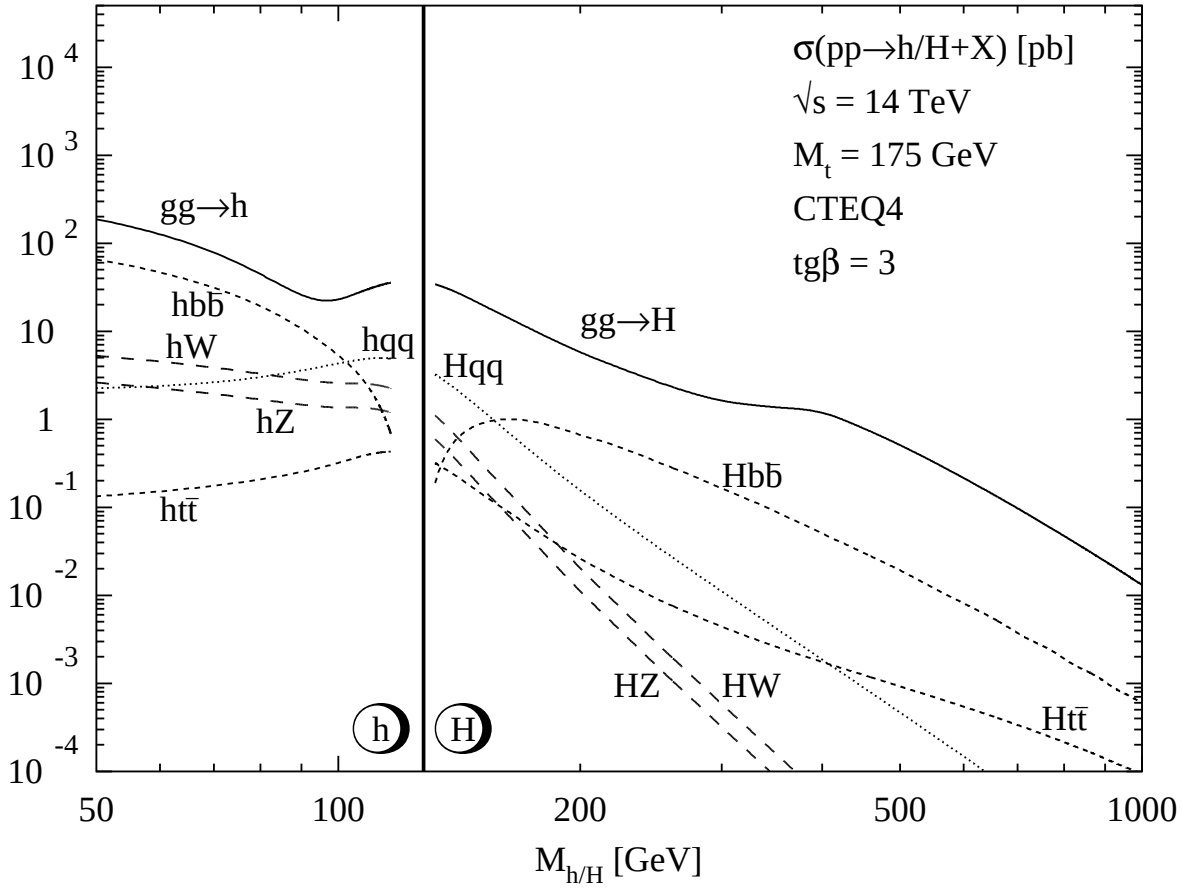
Dittmaier, Krämer, S.

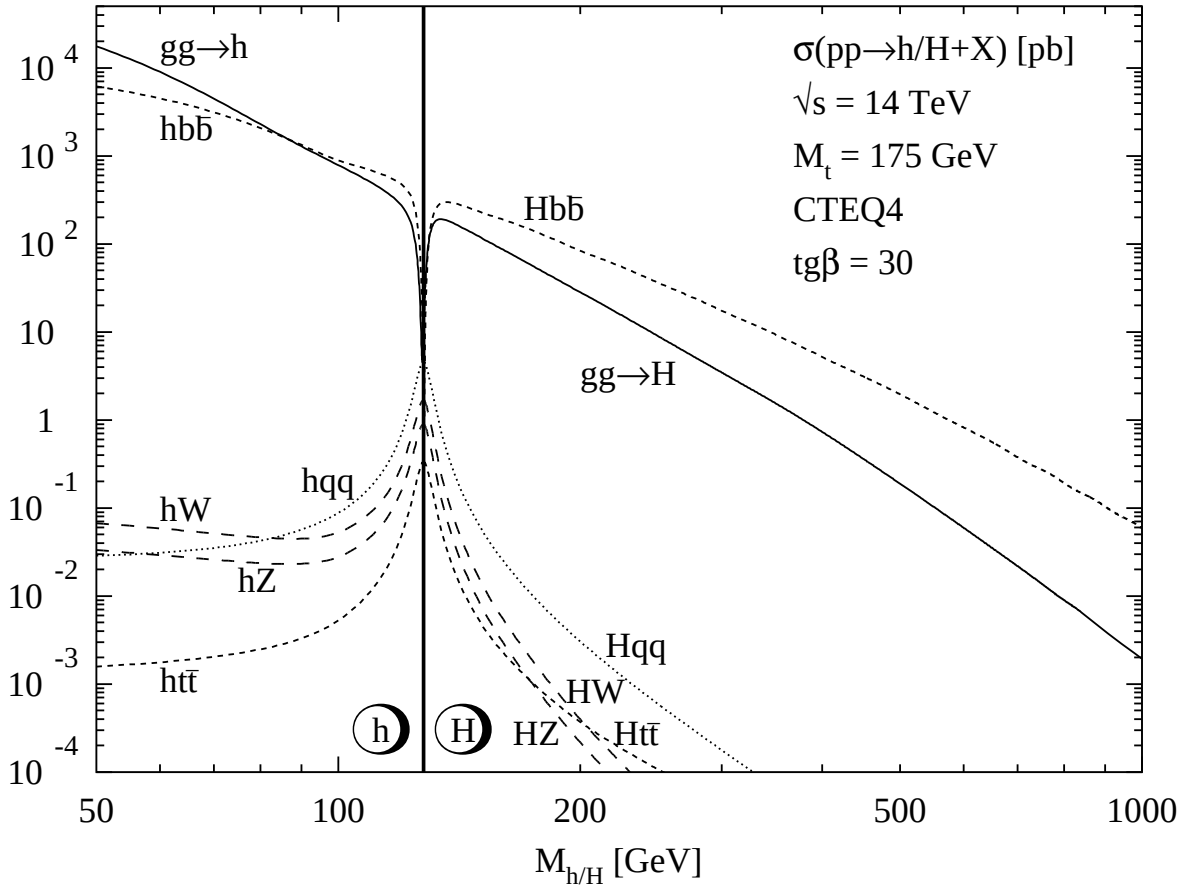

σ(pp̄→h/H+X) [pb]
√s = 2 TeV
M_t = 175 GeV
CTEQ4
tgβ = 6
gg→h
hbb̄
hW
hZ
hqq
htt̄
gg→H
HW
Hbb̄
HZ
Hqq
Htt̄
h
H
M_h/H [GeV]

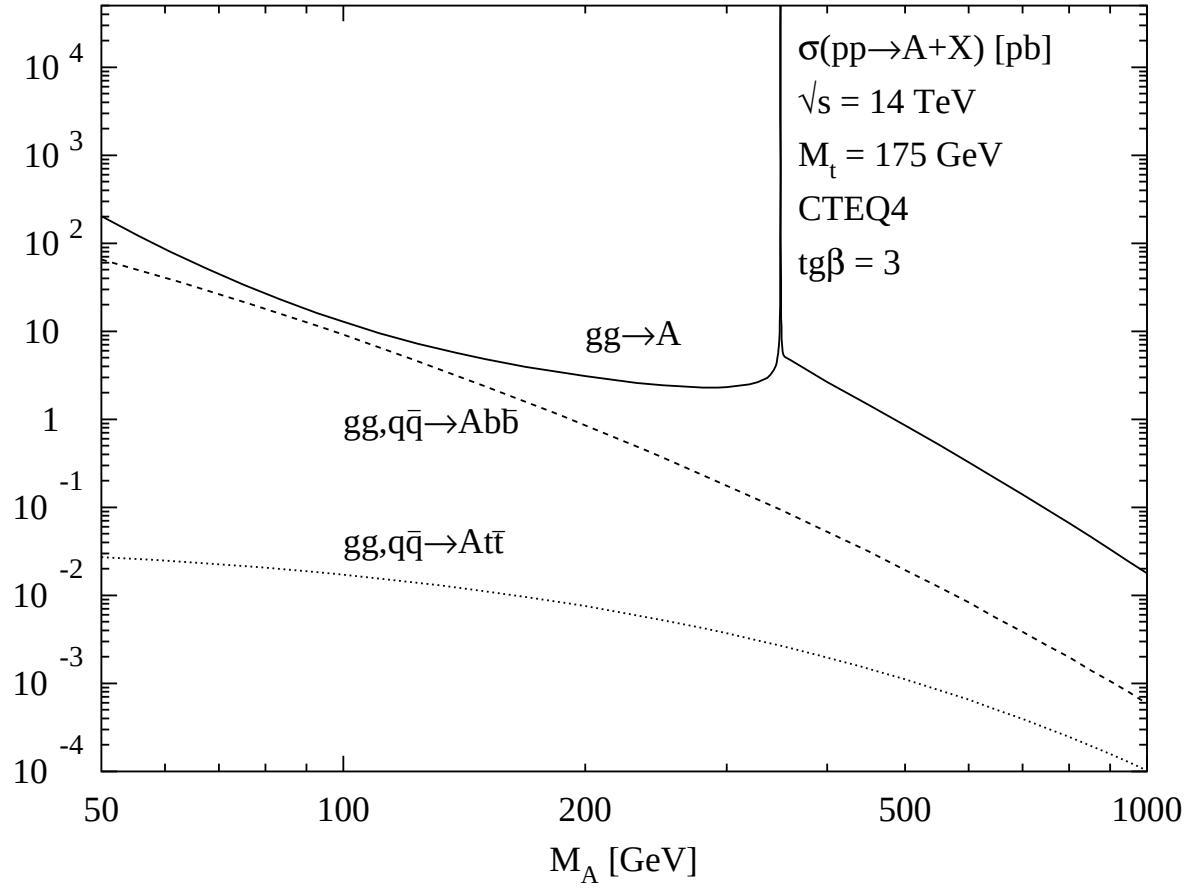

σ(pp̄→h/H+X) [pb]
√s = 2 TeV
M_t = 175 GeV
CTEQ4
tgβ = 30
gg→h
hbb̄
hW
hZ
hqq
htt̄
gg→H
Hbb̄
HW
HZ
Hqq
Htt̄
h
H
M_h/H [GeV]

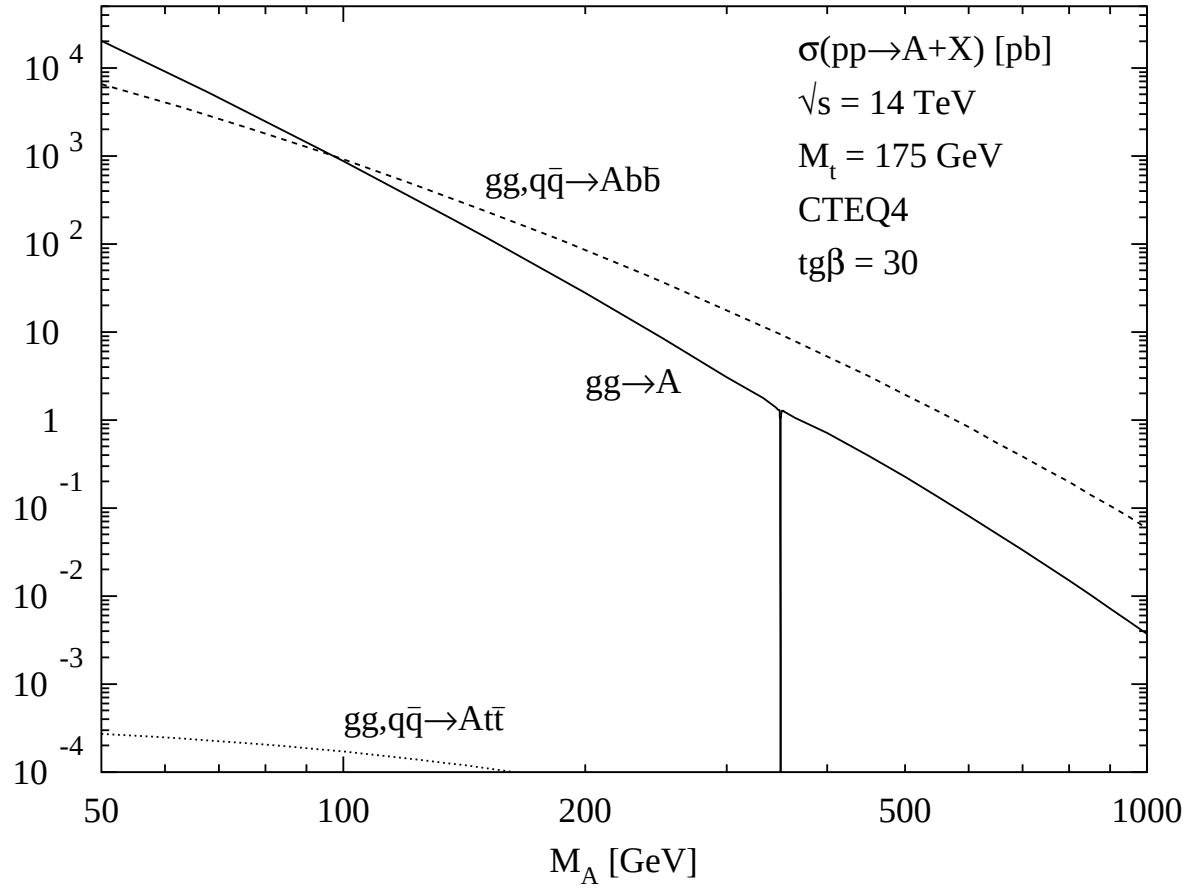

σ(pp̄→A+X) [pb]
√s = 2 TeV
M_t = 175 GeV
CTEQ4
tgβ = 6
gg→A
gg,qq̄→Abb̄
1
10^-1
10^-2
10^-3
80
100
120
140
160
180
200
M_A [GeV]


σ(pp̄→A+X) [pb]
√s = 2 TeV
M_t = 175 GeV
CTEQ4
tgβ = 30
gg→A
gg,qq̄→Abb̄
10
1
10^-1
80
100
120
140
160
180
200
M_A [GeV]


σ(pp→h/H+X) [pb]
√s = 14 TeV
M_t = 175 GeV
CTEQ4
tgβ = 3
gg→h
hbb̄
hW
hqq
hZ
htt̄
gg→H
Hqq
Hbb̄
HZ
HW
Htt̄
h
H
M_h/H [GeV]

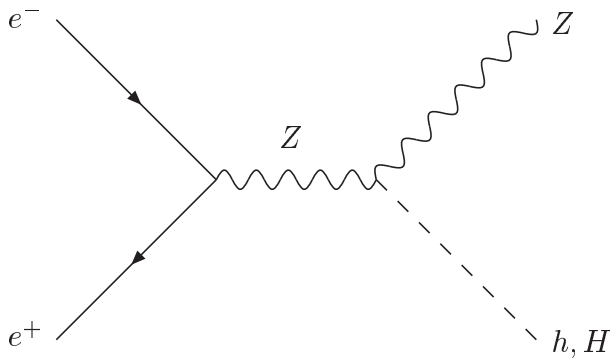

σ(pp→h/H+X) [pb]
√s = 14 TeV
M_t = 175 GeV
CTEQ4
tgβ = 30
gg→h
hbb̄
Hbb̄
gg→H
hqq
hW
hZ
htt̄
Hqq
HW
HZ
Htt̄
h
H
M_h/H [GeV]


σ(pp→A+X) [pb]
√s = 14 TeV
M_t = 175 GeV
CTEQ4
tgβ = 3
gg→A
gg,qq̄→Abb̄
gg,qq̄→Att̄
10^4
10^3
10^2
10
1
10^-1
10^-2
10^-3
10^-4
50
100
200
500
1000
M_A [GeV]


σ(pp→A+X) [pb]
√s = 14 TeV
M_t = 175 GeV
CTEQ4
tgβ = 30
gg,qq̄→Abb̄
gg→A
gg,qq̄→Att̄
10^4
10^3
10^2
10
1
10^-1
10^-2
10^-3
10^-4
50
100
200
500
1000
M_A [GeV]

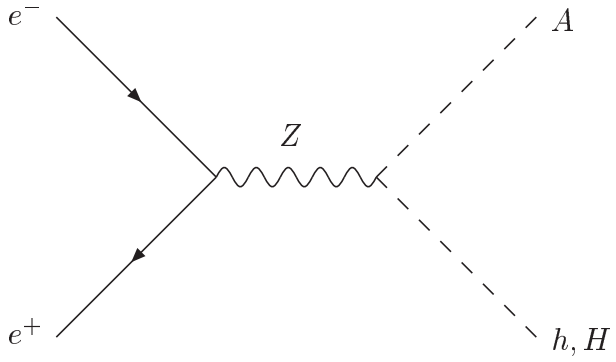
(b) $\underline{e^+e^-}$ colliders

- Higgs–strahlung: $e^+e^- \to Z + h/H$



$\sigma_h \propto \sin^2(\beta - \alpha)$
$\sigma_H \propto \cos^2(\beta - \alpha)$
electroweak corrections: $\sim -5 \ldots -10\%$

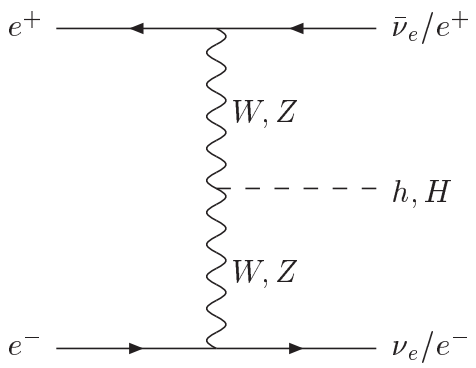
- pair production: $e^+e^- \to A + h/H$



$\sigma_h \propto \cos^2(\beta - \alpha)$
$\sigma_H \propto \sin^2(\beta - \alpha)$

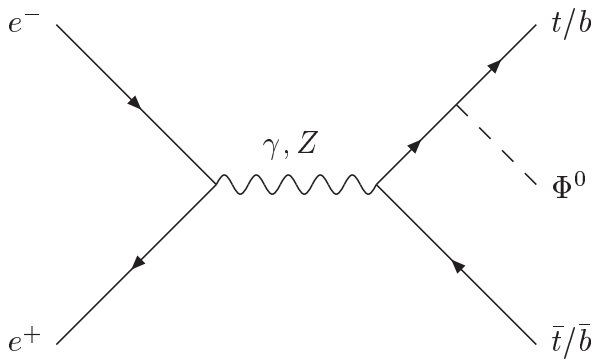
$\leftarrow$ complementarity

- W/Z fusion: $e^+e^- \to \nu_e\bar{\nu}_e/e^+e^- + h/H$

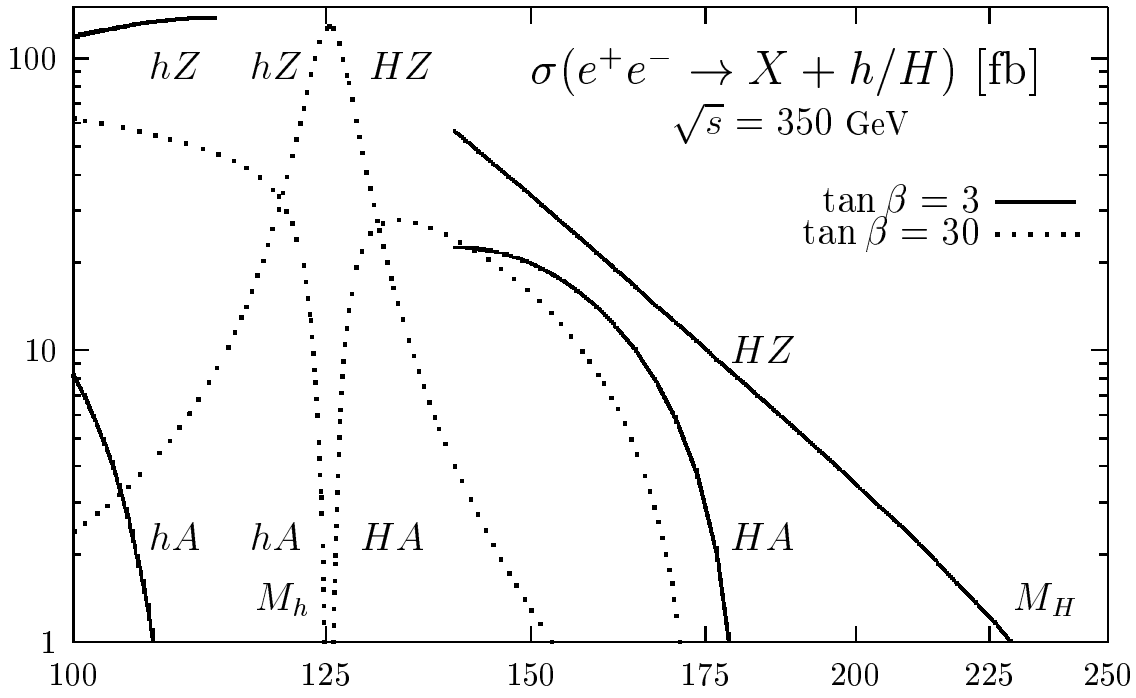


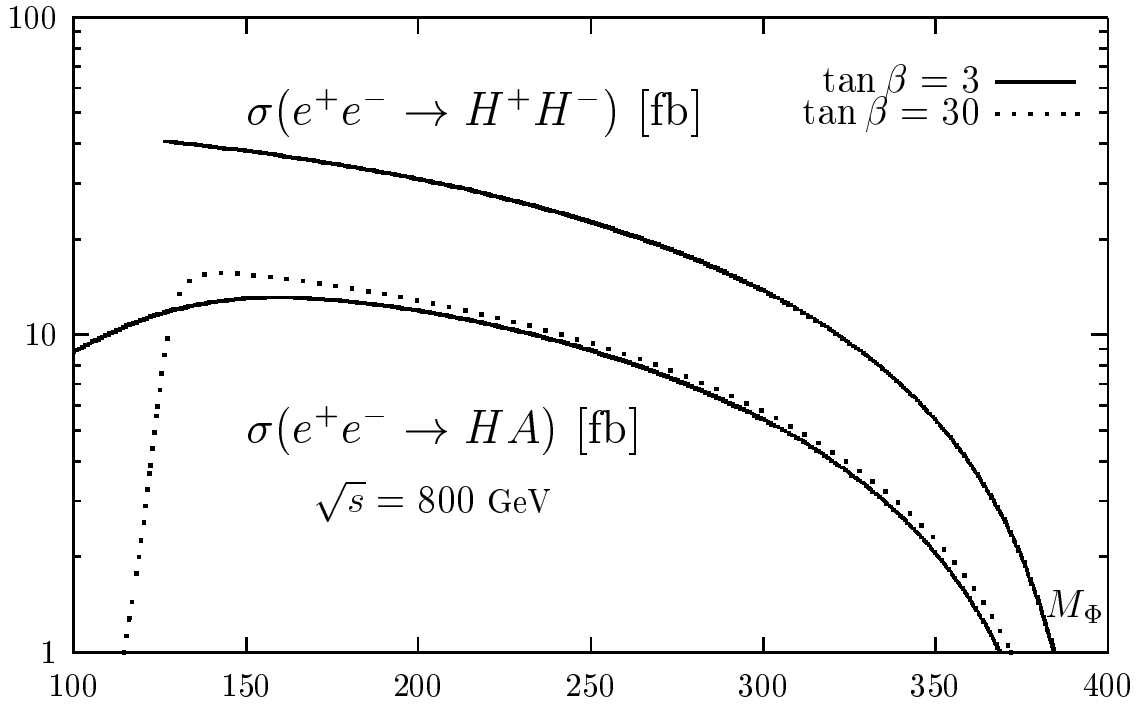
relevant at high energies
electroweak corrections: $\sim -2 \ldots -10\%$

- bremsstrahlung: $e^+e^- \to t\bar{t}/b\bar{b} + h/H/A$



$\leftarrow$ measurement of Yukawa couplings
QCD corrections: $\sim -10 \ldots +50\%$
electroweak corrections: $\sim +10\%$





Djouadi, Kalinowski, Zerwas