§4. SUSY–GUT

SUSY: physically motivated by GUT: stability of radiative corrections to W, Z, H masses
$\leftarrow$ reconstruction of fundamental supersymmetric theory at GUT/PLANCK–scale

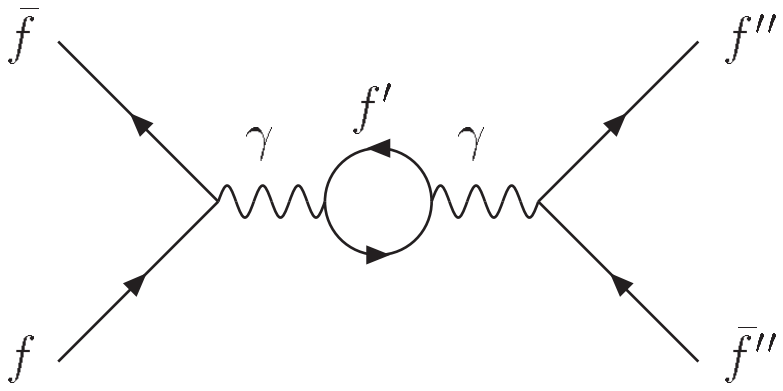
MSSM: 124 free parameters $\leftarrow$ constrained due to unification at GUT scale

(a) unification of gauge couplings

SM:
$SU_3 \times SU_2 \times U_1 \underset{\text{GUT}}{\rightarrow} SU_5$

$g_3 \quad g_2 \quad g_1 \rightarrow g_5$: $\sin^2\theta_W = \dfrac{\langle I_3^2\rangle}{\langle Q^2\rangle} = \dfrac{3}{8}$ @ GUT

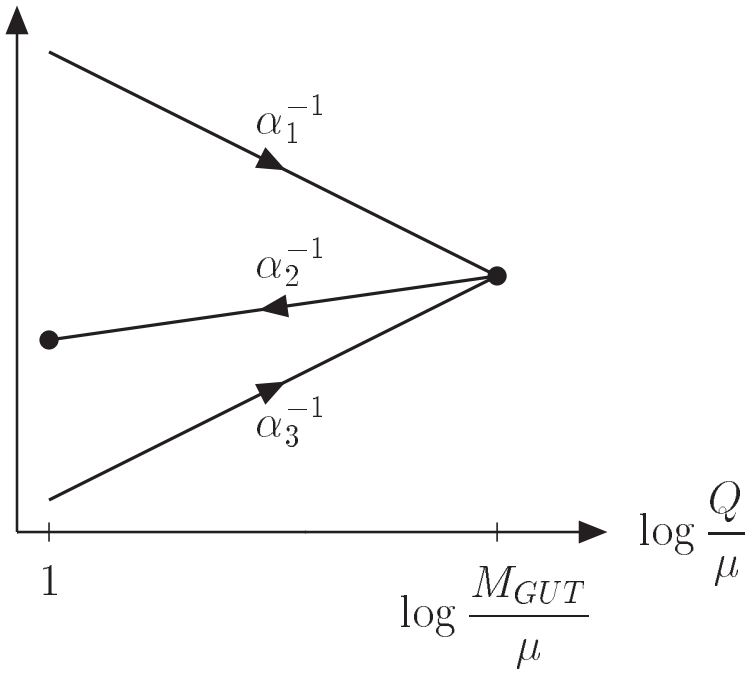
running couplings:



$$\frac{\partial \alpha_i}{\partial \log \mu^2} = -\frac{b_i}{4\pi}\alpha_i^2 + \mathcal{O}(\alpha_i^3)$$

SM:	0		4/3		1/10
	22/3	$-N_G$	4/3	$-N_H$	1/6
	11		4/3		0

SUSY:	0		2		3/10
	6	$-N_G$	2	$-N_H$	1/2
	9		2		0



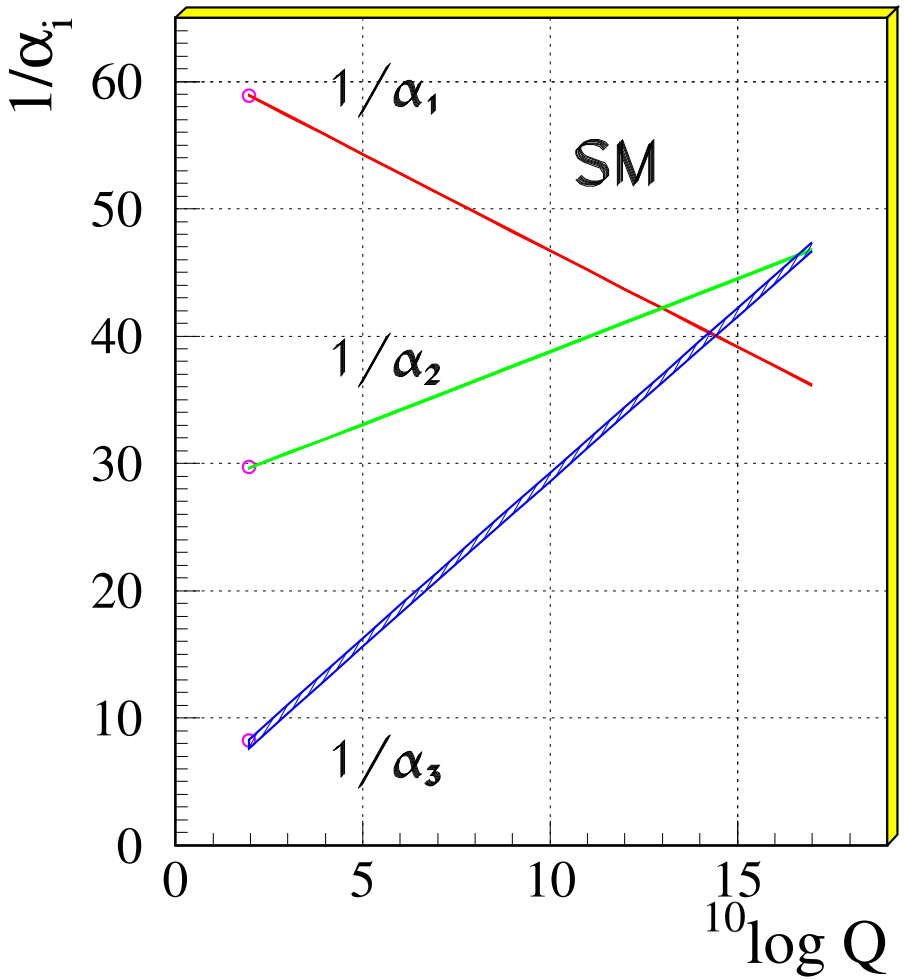
$\underline{\sin^2 \theta_W(M_Z^2)}$:

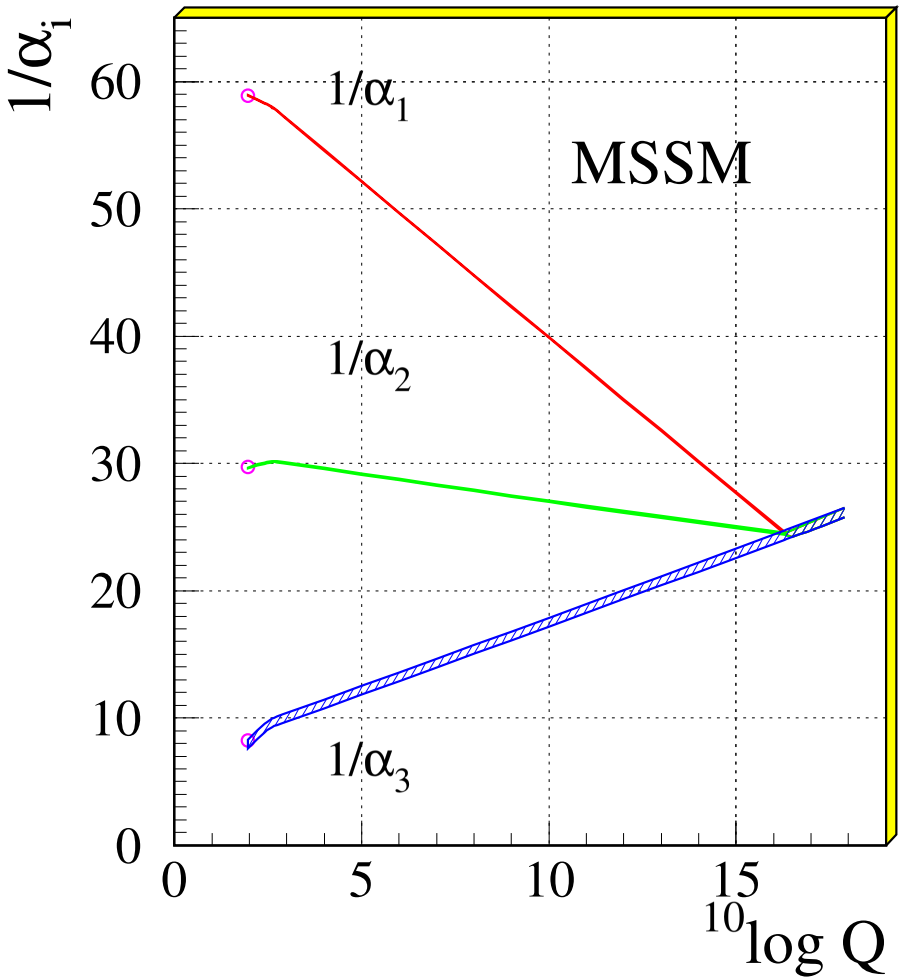
SM:	0.2100 ± 0.0026
MSSM:	0.2335 ± 0.0017
exp:	0.2316 ± 0.0002

• SM couplings cannot be unified, SUSY couplings can

• SUSY prediction of $\sin^2 \theta_W$ within 2‰. Independent of details of $\tilde{M} \sim 1$ TeV spectrum, only sensitive on particle spectrum

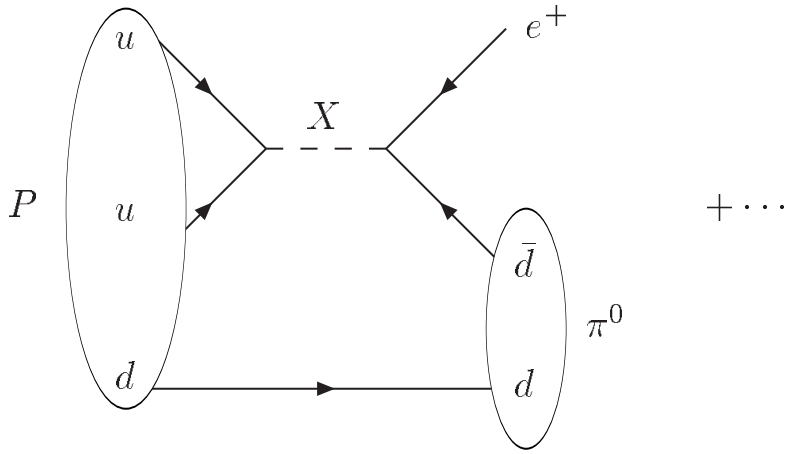
Unification of the Coupling Constants in the SM and the minimal MSSM





(b) proton decay:

SM–SU(5): gauge interactions between leptons and quarks:

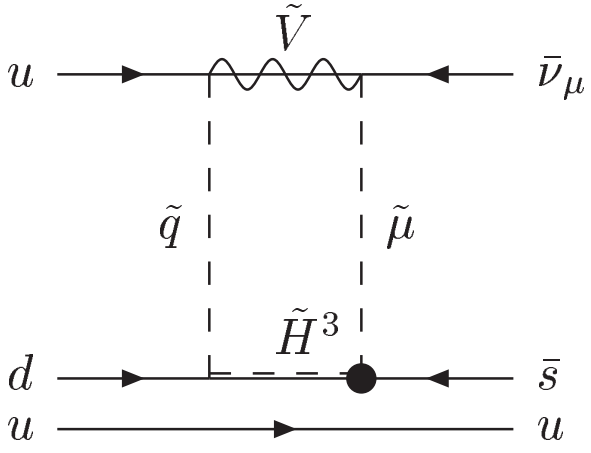


$$\tau(P \to \pi^0 e^+) \sim \frac{M_X^4}{\alpha_5^2 M_P^5}$$

SM: $M_X \sim 10^{15}$ GeV: $\sim 10^{30}$ a

SUSY: $M_X > 10^{16}$ GeV: $\gtrsim 10^{35}$ a [exp.: $> 2.4{*}10^{34}$a]

but dim 5 contribution:



$$\tau(P \to K^+ \bar{\nu}_\mu) \sim \left(\frac{16\pi^2}{\lambda^2 g^2}\right)^2 \frac{m_{\tilde{H}^3}^2 \tilde{m}^2}{M_P^5}$$

close to experimental limit

(c) RADIATIVE SYMMETRY BREAKING

universal sGUT masses: [mSUGRA]

gauginos: $M_1, M_2, M_3 = M_{1/2}$
scalars: $M_H^2 - \mu^2 = m_{\tilde{\ell}}^2 = m_{\tilde{q}}^2 = M_0^2$

RGE: GUT $\to$ ELW

$$\frac{\partial}{\partial \log Q}\begin{bmatrix} M_{H_2}^2 \\ m_{\tilde{t}_R}^2 \\ m_{\tilde{t}_L}^2 \end{bmatrix} = \frac{g_t^2}{8\pi^2}\begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} M_{H_2}^2 \\ m_{\tilde{t}_R}^2 \\ m_{\tilde{t}_L}^2 \end{bmatrix} + \frac{g_t^2 A_t^2}{8\pi^2}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

evolution: $M_{H_2}^2(Q^2) \approx M_0^2 + \mu^2 - \frac{3g_t^2}{8\pi^2}(3M_0^2 + \mu^2)\log\frac{M_{GUT}}{Q}$

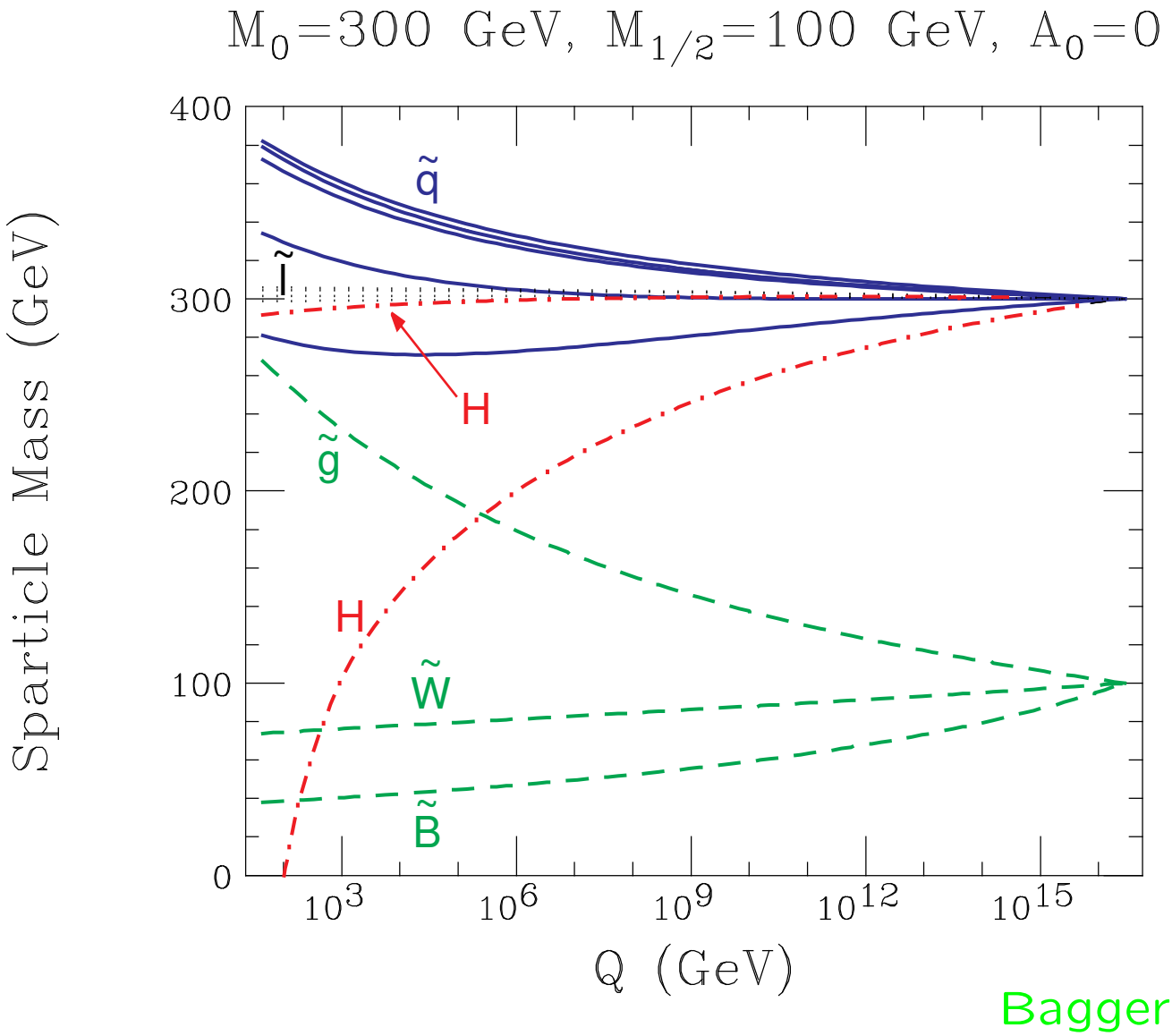
$M_{H_2}^2(M_Z^2) < 0$ possible for $m_t \sim 100 - 200$ GeV
$\to$ radiative symmetry breaking $SU_3 \times SU_2 \times U_1 \to SU_3 \times U_{1em}$

m_t small: no SSB $\qquad$ m_t large: SSB of SU_3, U_{1em}

• LO: $\frac{M_Z^2}{2} = \frac{M_{H_1}^2 - M_{H_2}^2 \text{tg}^2\beta}{\text{tg}^2\beta - 1} - |\mu|^2 \qquad [M_{H_i} = m_i]$

$$2B\mu = (M_{H_1}^2 + M_{H_2}^2 + 2|\mu|^2)\sin 2\beta$$

adjustment: $M_Z = \frac{\sqrt{g^2 + g'^2}}{2}v \qquad \Rightarrow |\mu|$ fixed



(d) FERMION MASSES

SU_5: $m_b = m_\tau$ @ GUT $\rightarrow$ exp. mass at elw. scale

RGE

(Top)

$\Longrightarrow$ 2 solutions: $\mathrm{tg}\beta \sim 2 \pm 1$ [LEP2!]

$\mathrm{tg}\beta \sim 20 - 50$

for given top mass

§5. SUSY Breaking Mechanisms

Ferrara SR: no spontaneous SUSY breaking in eigenworld: $\sum(-1)^{2J}(2J+1)M_J^2 \neq 0$

mechanisms:	supergravity	mSUGRA ⟵
	gauge mediated SB	GMSB ⟵
	anomaly med. SB	AMSB
	Scherk–Schwarz SB	SSSB ...

minimal SUPERGRAVITY:

hidden sector SUSY: $\langle\bar{\lambda}\lambda\rangle \neq 0$	⟿ $GRAV.$ ⟿	eigenworld

non-renormalizable gravitational interaction $\rightarrow$ F-condensates of non–MSSM fields Z that couple to MSSM:

$$\frac{Z^\dagger Z}{M_{Planck}^2}\Phi^\dagger\Phi \rightarrow \frac{|\langle F_Z\rangle|^2}{M_{Planck}^2}\phi^*\phi$$

$$\frac{Z}{M_{Planck}}\Phi^3 \rightarrow \frac{\langle F_Z\rangle}{M_{Planck}}\phi^3$$

scale: $M_{soft}^2 \sim \frac{M^4[\cancel{SUSY}]}{M_{Planck}^2} \sim 1$ TeV^2

$\Rightarrow$ $\boxed{M[\cancel{SUSY}] \sim 10^{11}\ \text{GeV}}$

soft parameters "universal": 5 parameters:

$$\underline{\underline{M_0, M_{1/2}, A_0, \text{tg}\beta, \text{sgn}\mu}}$$

<u>evolution:</u> GUT $\to$ ELW:

$$\begin{aligned} \frac{\partial M_i}{\partial \log \mu^2} &= -\frac{b_i}{4\pi}\alpha_i M_i \\ \frac{\partial \alpha_i}{\partial \log \mu^2} &= -\frac{b_i}{4\pi}\alpha_i^2 \\ \Rightarrow \frac{M_i}{\alpha_i} &= \text{const} \end{aligned}$$

gaugino masses run as gauge couplings

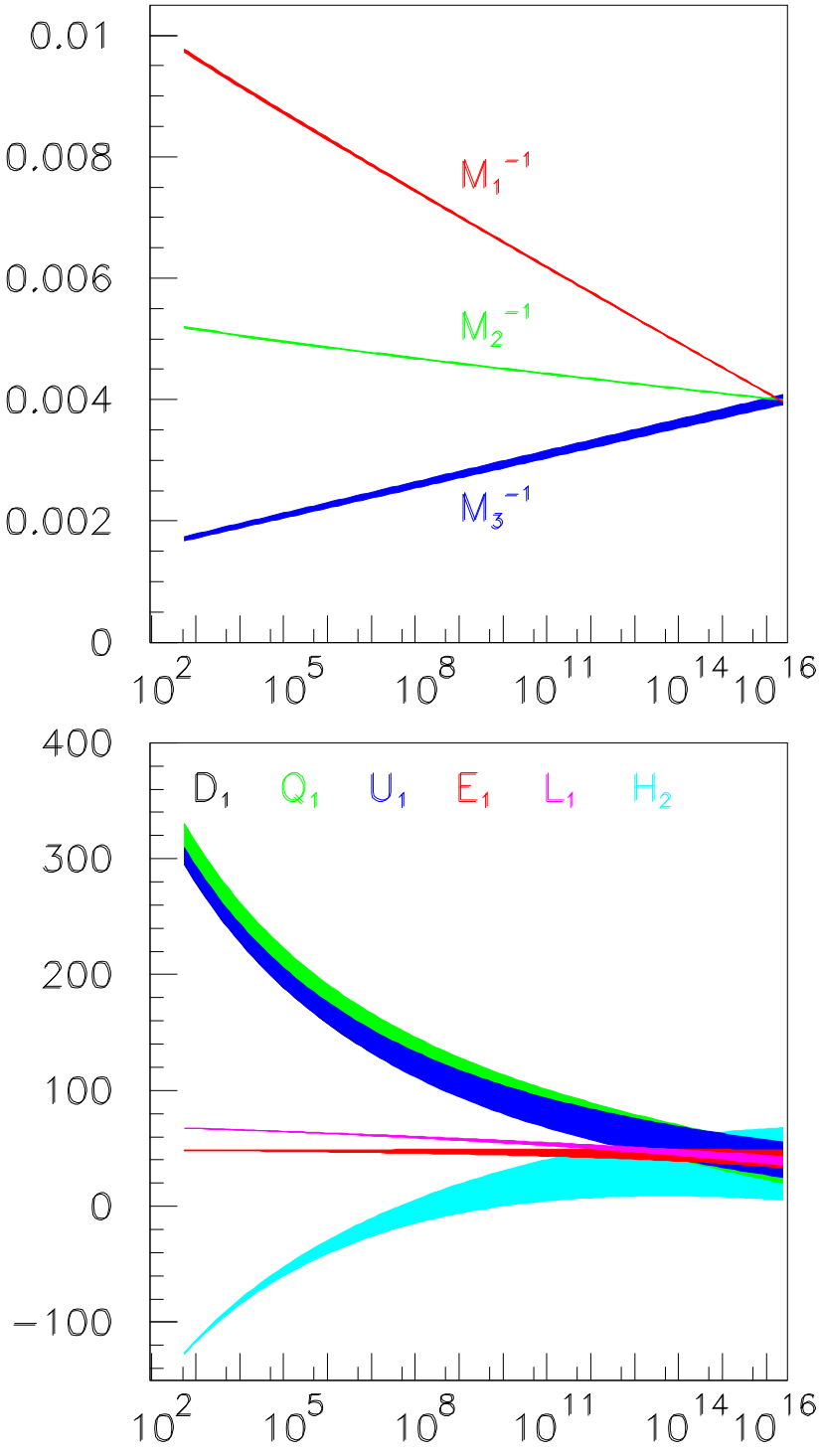
$$\Rightarrow M_1 = M_2 \cdot \frac{5}{3}\text{tg}^2\theta_W \sim \frac{1}{2}M_2 \quad [M_i(M_{GUT}) = M_{1/2}]$$

$$\frac{M_3}{M_2} = \frac{\alpha_3}{\alpha_2} \gg 1 \quad \Rightarrow \quad m_{\tilde{g}} \gg m_{\tilde{\chi}}$$

$$\begin{aligned} \frac{\partial m_{\tilde{\ell}}^2}{\partial \log \mu^2} &= c_\ell \frac{m_\ell^2}{v^2} m_{\tilde{\ell}}^2 \ldots \\ \frac{\partial m_{\tilde{q}}^2}{\partial \log \mu^2} &= c_q \frac{m_q^2}{v^2} m_{\tilde{q}}^2 - c_3 \alpha_s M_3^2 \ldots \end{aligned}$$

slepton masses increase less than squark masses: $m_{\tilde{q}} \sim (2-3) \times m_{\tilde{\ell}}$

reconstruction of fundamental theory at GUT/Planck scale from low-energy data (LHC, LC)



Blair, Porod, Zerwas

sfermion masses: [approximate formulae]

(i) 1. and 2. generation: degenerate

$$\begin{aligned}
m^2_{\tilde{u}_L} &= M_0^2 + 6.28 M_{1/2}^2 + 0.35 M_Z^2 \cos 2\beta \\
m^2_{\tilde{d}_L} &= M_0^2 + 6.28 M_{1/2}^2 - 0.42 M_Z^2 \cos 2\beta \\
m^2_{\tilde{u}_R} &= M_0^2 + 5.87 M_{1/2}^2 + 0.16 M_Z^2 \cos 2\beta \\
m^2_{\tilde{d}_R} &= M_0^2 + 5.87 M_{1/2}^2 - 0.08 M_Z^2 \cos 2\beta \\
m^2_{\tilde{\nu}_L} &= M_0^2 + 0.52 M_{1/2}^2 + 0.50 M_Z^2 \cos 2\beta \\
m^2_{\tilde{e}_L} &= M_0^2 + 0.52 M_{1/2}^2 - 0.27 M_Z^2 \cos 2\beta \\
m^2_{\tilde{e}_R} &= M_0^2 + 0.15 M_{1/2}^2 - 0.23 M_Z^2 \cos 2\beta
\end{aligned}$$

$$\Rightarrow \qquad m_{\tilde{q}_L} > m_{\tilde{q}_R} > m_{\tilde{\ell}_L} > m_{\tilde{\ell}_R}$$

$$m^2_{\tilde{e}_L} - m^2_{\tilde{\nu}_L} = m^2_{\tilde{d}_L} - m^2_{\tilde{u}_L} = -M_W^2 \cos 2\beta$$

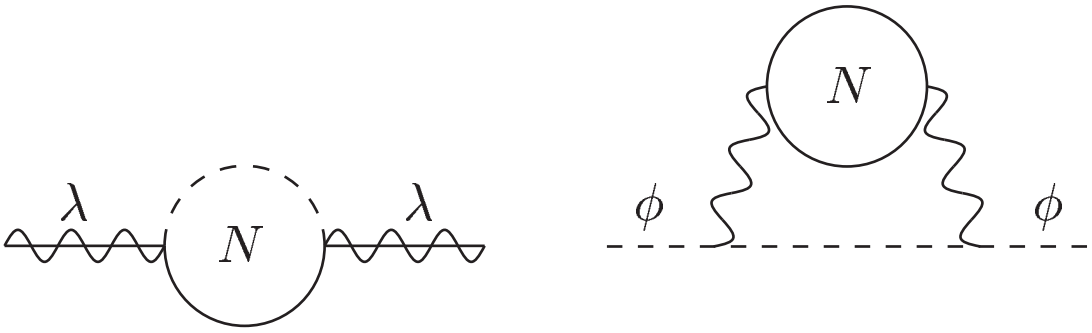
(ii) 3. generation: Yukawa couplings important

$$\begin{aligned}
m^2_{\tilde{t}_L} &= 0.52 M_0^2 + 5.4 M_{1/2}^2 + 0.35 M_Z^2 \cos 2\beta \\
m^2_{\tilde{b}_L} &= 0.52 M_0^2 + 5.4 M_{1/2}^2 - 0.42 M_Z^2 \cos 2\beta \\
m^2_{\tilde{t}_R} &= 0.30 M_0^2 + 3.8 M_{1/2}^2 + 0.16 M_Z^2 \cos 2\beta \\
m^2_{\tilde{b}_R} &= 0.90 M_0^2 + 3.8 M_{1/2}^2 - 0.08 M_Z^2 \cos 2\beta
\end{aligned}$$

GMSB:

• $\langle F \rangle \ll 10^{22}\text{GeV}^2 \Rightarrow$ suppression of SUGRA effects

• assumptions: N messenger fields with mass M_{mess} and SM quantum numbers $\Rightarrow$ mass generation through quantum fluctuations:



$$M_i = N \frac{g_i^2(M_{mess}^2)}{16\pi^2} \frac{\langle F \rangle}{M_{mess}}$$

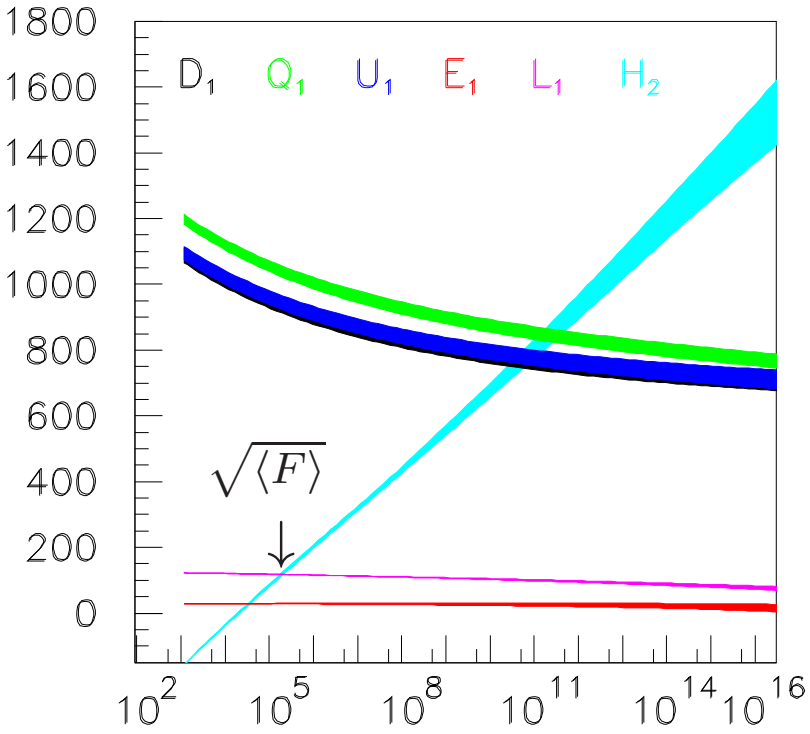
$$m_{\tilde{f}}^2 = 2N \sum_i C_{if} \left[\frac{g_i^2(M_{mess}^2)}{16\pi^2}\right]^2 \left[\frac{\langle F \rangle}{M_{mess}}\right]^2$$

• gaugino masses evolve analogous to mSUGRA, but without unification

• scalar masses significantly different: determined by $\langle F \rangle, M_{mess}, N$

• 5 parameters: $\underline{\underline{\langle F \rangle, M_{mess}, N, \text{tg}\beta, \text{sgn}\mu}}$

• $m_{\tilde{e}_L} = M_{H_u}$ for $\mu = \sqrt{\langle F \rangle} \rightarrow$ experimental reconstruction of messenger scale $\sqrt{\langle F \rangle} = M[\cancel{SUSY}]$



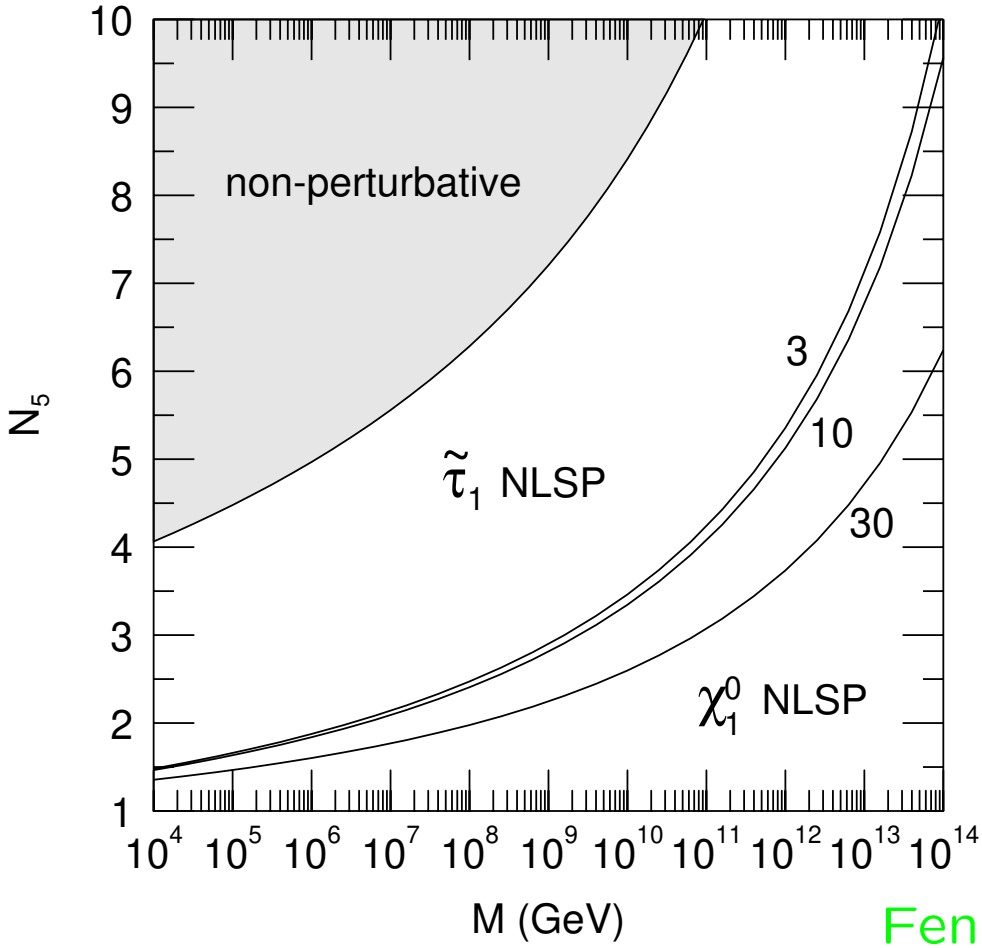
Blair, Porod, Zerwas

• lower bound on M_{mess} and SUSY masses $\Rightarrow \sqrt{\langle F \rangle} \gtrsim 10^5$ GeV

• SUGRA $\downarrow \Rightarrow \sqrt{\langle F \rangle} \lesssim 10^9$ GeV

• central consequence: $m_{\tilde{G}} \sim \dfrac{\langle F \rangle}{M_{Planck}} \ll M_{weak} \Rightarrow$ gravitino is LSP!

• NLSP: (i) N small $\rightarrow$ neutralino
(ii) N large $\rightarrow$ slepton



$\leftarrow \mathrm{tg}\beta$

Feng, Moroi

• decay: $NLSP \to \tilde{G} + X \to$ significant flight distance

$$L \sim 0.1mm \times \sqrt{\frac{E_{NLSP}^2 - m_{NLSP}^2}{m_{NLSP}}} \left[\frac{\sqrt{\langle F \rangle}}{10^5 \text{ GeV}}\right]^4 \left[\frac{100 \text{ GeV}}{m_{NLSP}}\right]^5$$

$\langle F \rangle$ small: decay within the detector $\to$ energetic photons and leptons

$\langle F \rangle$ large: NLSP flies macroscopic distances $\to$
(i) $\not{E}_T$, if $NLSP = \tilde{\chi}^0$
(ii) anomalous charged particle tracks, if $NLSP = \tilde{\ell}$