§3. SUSY Standard Model

SM alone cannot be formulated as SUSY theory $[N \leq 8]$

$\Rightarrow$ $\boxed{\text{SUSY Standard Model} = SM \otimes SUSY[N = 1]}$
(minimal)

$\rightarrow$ doubling of particle spectrum: SM + SUSY partner

minimal model MSSM:

VECTOR MULTIPLETS

$J = 1$	$J = \frac{1}{2}$
gluon g	gluino $\tilde{g}$
$W^{\pm}, W^3$	wino $\tilde{W}^{\pm}, \tilde{W}^3$
B	bino $\tilde{B}$

CHIRAL MULTIPLETS

$J = \frac{1}{2}$	$J = 0$
quarks q_L, q_R	squarks $\tilde{q}_L, \tilde{q}_R$
leptons ℓ_L, ℓ_R	sleptons $\tilde{\ell}_L, \tilde{\ell}_R$
higgsinos $\tilde{H}_1, \tilde{H}_2$	Higgs H_1, H_2

Higgs sector:

SM: $\mathcal{L}_H = g_d(\bar{d}_L\phi)d_R + g_u(\bar{u}_L\phi^c)u_R + h.c.$

$$\phi \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$\phi^c = i\sigma^2\phi^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v+H \\ 0 \end{pmatrix}$$

MSSM: superpotential holomorphic $\to$ 2 Higgs doublets for up– and down–type fermions

anomaly freedom: $TrQ_{\tilde{H}} = 0$

Lagrange density:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_D + \mathcal{L}_W + \mathcal{L}_{soft} \\ \mathcal{L}_{gauge} &= -\frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \\ &+ Tr[\bar{\tilde{g}}i\not{D}\tilde{g}] + Tr[\overline{\tilde{W}}i\not{D}\tilde{W}] + \frac{1}{2}\overline{\tilde{B}}i\not{\partial}\tilde{B} \end{aligned}$$

$$
\begin{aligned}
\mathcal{L}_{matter} &= \sum_{\psi=f,\tilde{H}_i} \bar{\psi} i \not{D} \psi + \sum_{\tilde{f},H_i} |D_\mu \phi|^2 \\
&+ i \sum_{\psi,\phi,V} \frac{g_V}{\sqrt{2}} \left[\overline{\psi_L} T^a \tilde{V}^a \phi - \tilde{V}^a T^a \psi_L \phi^* \right] \\
\mathcal{L}_D &= -\frac{1}{2} \sum_{a,V} |D_a^V|^2 \quad \text{with} \quad D_a^V = -g_V \phi_i^* T_{ij}^a \phi_j
\end{aligned}
$$

[Higgs fields,. . .]

$$
\begin{aligned}
\mathcal{L}_W &= -\sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 - \frac{1}{2} \sum_{ij} \overline{{\psi_{iL}}^c} \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \psi_{jL} + h.c. \\
W &= W_R + W_{\not R} \qquad \left[(\psi^c)^c = \psi;\ \overline{\psi_1^c} \psi_2^c = \overline{\psi_2} \psi_1 \right] \\
W_R &= -\epsilon_{ij} \mu H_1^i H_2^j \\
&+ \epsilon_{ij} \left[\lambda_L H_1^i \tilde{L}_j \tilde{E}^c + \lambda_D H_1^i \tilde{Q}_j \tilde{D}^c + \lambda_U H_2^i \tilde{Q}_j \tilde{U}^c \right] \\
W_{\not R} &= \epsilon_{ij} \left[\lambda \tilde{L}_i \tilde{L}_j \tilde{E}^c + \lambda' \tilde{L}_i \tilde{Q}_j \tilde{D}^c \right] + \lambda'' \tilde{U}^c \tilde{D}^c \tilde{D}^c
\end{aligned}
$$

(1) mass generation in SM via Higgs mechanism
(2) R–parity violation: $\Delta L \neq 0, \Delta B \neq 0$; P decay $\rightarrow \lambda' \lambda'' \approx 0$

$$
\begin{aligned}
\mathcal{L}_{soft} &= -\frac{1}{2} \sum_{i=1}^{3} M_i \bar{\lambda}_i^a \lambda_i^a - m_1^2 |H_1|^2 - m_2^2 |H_2|^2 \\
&+ B\mu \epsilon_{ij} (H_1^i H_2^j + h.c.) - M_{\tilde{Q}}^2 (\tilde{u}_L^\dagger \tilde{u}_L + \tilde{d}_L^\dagger \tilde{d}_L) \\
&- M_{\tilde{U}}^2 \tilde{u}_R^\dagger \tilde{u}_R - M_{\tilde{D}}^2 \tilde{d}_R^\dagger \tilde{d}_R + (\tilde{\ell}) \\
&- \frac{g}{\sqrt{2} M_W} \epsilon_{ij} \left[\frac{M_d}{\cos\beta} A_d H_1^i \tilde{Q}^j \tilde{d}_R^\dagger + (\tilde{u}) + (\tilde{\ell}) \right]
\end{aligned}
$$

1.) HIGGS SECTOR

Higgs potential:

$$
\begin{aligned}
V &= (m_1^2+\mu^2)|H_1|^2+(m_2^2+\mu^2)|H_2|^2 \\
&- B\mu\epsilon_{ij}\left(H_1^i H_2^j + h.c.\right) \\
&+ \frac{g^2+g'^2}{8}\left[|H_1|^2-|H_2|^2\right]^2+\frac{g^2}{2}|H_1^\dagger H_2|^2 \\
H_1 &= \begin{pmatrix} H_1^1 \\ \\ H_1^2 \end{pmatrix} = \begin{pmatrix} \phi_1^{0*} \\ \\ -\phi_1^- \end{pmatrix} \\
H_2 &= \begin{pmatrix} H_2^1 \\ \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ \\ \phi_2^0 \end{pmatrix}
\end{aligned}
$$

$\leftarrow$ quartic couplings $\sim$ gauge couplings

$\leftarrow$ diagonalisation:

$$
\begin{aligned}
H_1^1 &= \frac{1}{\sqrt{2}}\left(v_1 + H^0 c_\alpha - h^0 s_\alpha + iA^0 s_\beta - iG^0 c_\beta\right) \\
H_1^2 &= H^- s_\beta - G^- c_\beta \\
H_2^1 &= H^+ c_\beta + G^+ s_\beta \\
H_2^2 &= \frac{1}{\sqrt{2}}\left(v_2 + H^0 s_\alpha + h^0 c_\alpha + iA^0 c_\beta + iG^0 s_\beta\right)
\end{aligned}
$$

$G^\pm, G^0 \Rightarrow$ Goldstone bosons

← parameter:

$$\begin{aligned} M_Z^2 &= \frac{g^2+g'^2}{4}(v_1^2+v_2^2) \\ M_A^2 &= B\mu\frac{v_1^2+v_2^2}{v_1 v_2} \\ \mathrm{tg}\beta &= \frac{v_2}{v_1} \\ m_1^2+\mu^2 &= M_A^2 s_\beta^2 - \frac{M_Z^2}{2}c_{2\beta} \\ m_2^2+\mu^2 &= M_A^2 c_\beta^2 + \frac{M_Z^2}{2}c_{2\beta} \end{aligned}$$

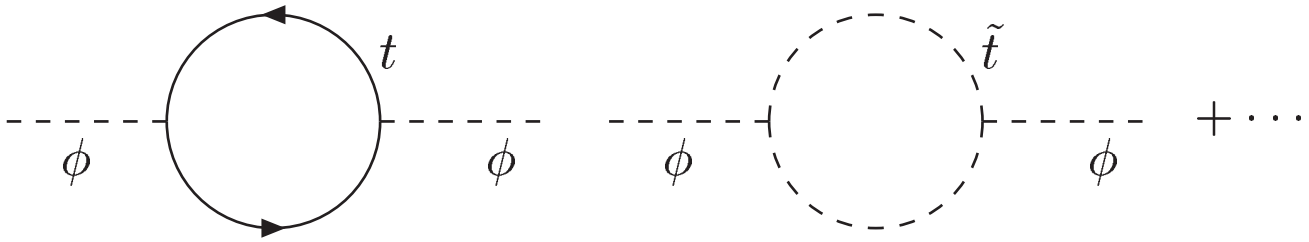
bilinear terms in V:

$$\begin{aligned} \mathcal{M}_\pm^2 &= \frac{1}{2}(M_A^2+M_W^2)s_{2\beta}\begin{pmatrix} \mathrm{tg}\beta & 1 \\ 1 & \cot\beta \end{pmatrix} \\ &\Rightarrow M_{H^\pm}^2 = M_A^2+M_W^2 \qquad \text{charged} \\ \mathcal{M}_a^2 &= \frac{1}{2}M_A^2 s_{2\beta}\begin{pmatrix} \mathrm{tg}\beta & 1 \\ 1 & \cot\beta \end{pmatrix} \Rightarrow M_A^2 \qquad \mathcal{CP}_o \\ \mathcal{M}_s^2 &= \frac{1}{2}s_{2\beta}\begin{pmatrix} M_A^2\mathrm{tg}\beta + M_Z^2\cot\beta & -M_A^2-M_Z^2 \\ -M_A^2-M_Z^2 & M_A^2\cot\beta + M_Z^2\mathrm{tg}\beta \end{pmatrix} \\ \Rightarrow M_{h,H}^2 &= \frac{1}{2}\left\{M_A^2+M_Z^2 \mp \sqrt{(M_A^2+M_Z^2)^2 - 4M_A^2M_Z^2c_{2\beta}^2}\right\} \qquad \mathcal{CP}_e \\ \mathrm{tg}2\alpha &= \mathrm{tg}2\beta\,\frac{M_A^2+M_Z^2}{M_A^2-M_Z^2} \qquad -\frac{\pi}{2} < \alpha < 0 \end{aligned}$$

<u>relations:</u>

$$\begin{aligned} M_h &< M_Z, M_A \\ M_H &> M_Z, M_A \\ M_{H^\pm} &> M_A, M_W \end{aligned}$$

modified by radiative corrections



$$\epsilon = \frac{3G_F m_t^4}{\sqrt{2}\pi^2 s_\beta^2} \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \; + \; \text{stop mixing } [\mu, A_t]$$

$$\longrightarrow \boxed{\begin{array}{l} M_h \lesssim 130 \text{ GeV} \\ \\ M_{H,A,H^\pm} = \ldots v \ldots 1 \text{ TeV} \end{array}}$$

2.) CHARGINOS/NEUTRALINOS

charginos: mixing of winos/higgsinos $\Rightarrow$ 2 eigenstates $\tilde{\chi}^\pm_{1/2}$ $\quad [H_u = H_2, H_d = H_1]$

$$\mathcal{L}_C \;=\; -\frac{1}{2}\psi_\pm^T \mathcal{M}_C \psi_\pm + h.c. \quad \text{with} \quad \psi_\pm = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \\ \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix}$$

$$\mathcal{M}_C \;=\; \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}$$

$$X = \begin{array}{c} \\ \\ \\ \end{array} \begin{array}{c} \mathcal{L}_{soft} \\ \left(\begin{array}{cc} M_2 & \sqrt{2} M_W s_\beta \\ \sqrt{2} M_W c_\beta & \mu \end{array} \right) \\ \mathcal{L}_{\lambda H \psi} \end{array} \begin{array}{c} \mathcal{L}_{\lambda H \psi} \\ \\ W(\Phi) \end{array}$$

physical states:

$$\chi_+ = \begin{pmatrix} \chi_1^+ \\ \chi_2^+ \end{pmatrix} = V \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix}$$

$$\chi_- = \begin{pmatrix} \chi_1^- \\ \chi_2^- \end{pmatrix} = U \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix}$$

$$[V^\dagger V = U^\dagger U = \mathbb{1}]$$

diagonalisation:

$$\mathcal{L}_C = -\frac{1}{2} \left\{ \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix}^T X \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix} + \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix}^T X^T \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix} \right\}$$

$$= -\frac{1}{2} \left\{ \chi_-^T U^* X V^{-1} \chi_+ + \chi_+^T V^* X^T U^{-1} \chi_- \right\}$$

$$[U^* = U^{\dagger T} = (U^{-1})^T]$$

$$\Longrightarrow U^* X V^{-1} = \begin{pmatrix} m_{\chi_1} & 0 \\ 0 & m_{\chi_2} \end{pmatrix}$$

$$\tilde{\chi}_i^+ = \begin{pmatrix} \chi_i^+ \\ (\chi_i^-)^\dagger \end{pmatrix} \qquad \text{[4 components]}$$

$$\mathcal{L}_C = -m_{\chi_1^\pm}\overline{\tilde{\chi}_1^+}\tilde{\chi}_1^+ - m_{\chi_2^\pm}\overline{\tilde{\chi}_2^+}\tilde{\chi}_2^+$$

$$m^2_{\chi^\pm_{1/2}} = \frac{1}{2}\Big\{M_2^2+\mu^2+2M_W^2 \mp\sqrt{(M_2^2-\mu^2)^2+4M_W^2(M_2^2+\mu^2+2M_2\mu s_{2\beta}+M_W^2c_{2\beta}^2)}\Big\}$$

limits: $\mu, M_2 \gg M_W$

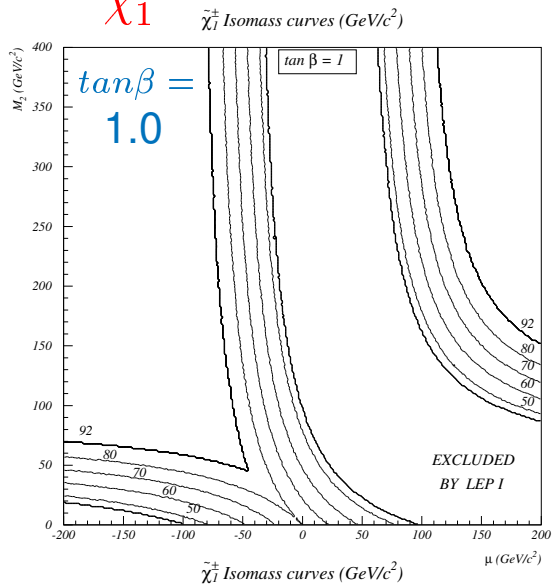
$$m_{\chi^\pm_{1/2}} = M_2 - \frac{M_W^2(M_2+\mu s_{2\beta})}{\mu^2-M_2^2}$$

$$m_{\chi^\pm_{2/1}} = |\mu| + \frac{M_W^2(|\mu|+\epsilon M_2 s_{2\beta})}{\mu^2-M_2^2} \qquad \epsilon = sign(\mu)$$

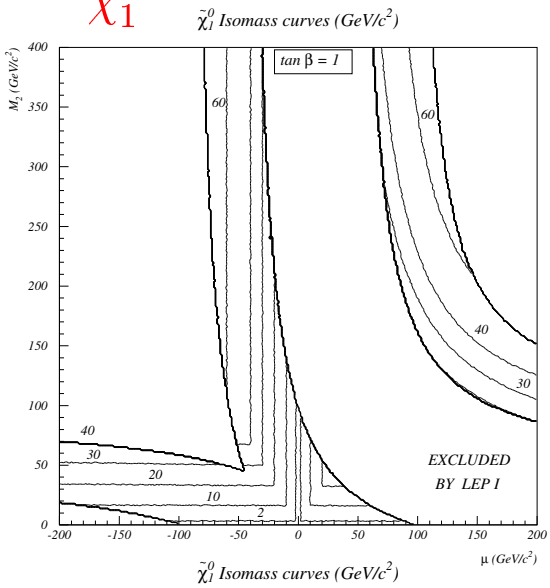
- $|\mu| \gg M_2$: $\tilde{\chi}_1^\pm$ gaugino–like, $\tilde{\chi}_2^\pm$ higgsino–like
- $|\mu| \ll M_2$: $\tilde{\chi}_1^\pm$ higgsino–like, $\tilde{\chi}_2^\pm$ gaugino–like
- $\mu \sim -M_2, \text{tg}\beta \sim 1$: $m_{\chi^\pm_{1/2}} \sim \sqrt{M_2^2+M_W^2}$ ∼degenerate

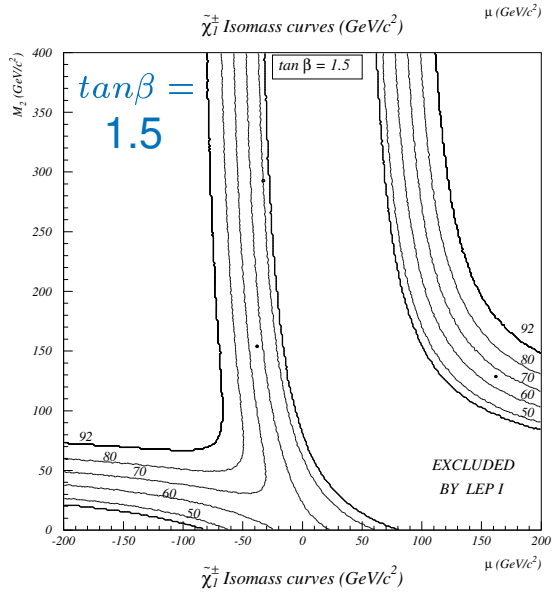
Chargino-Neutralino mass m_2 vs μ

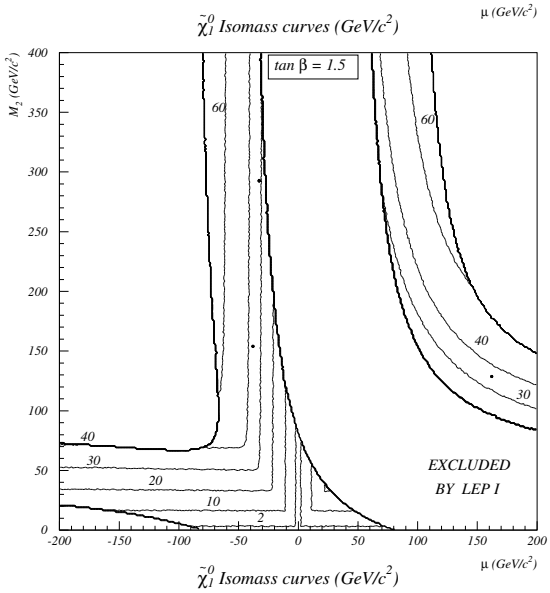
$\tilde{\chi}_1^{\pm}$

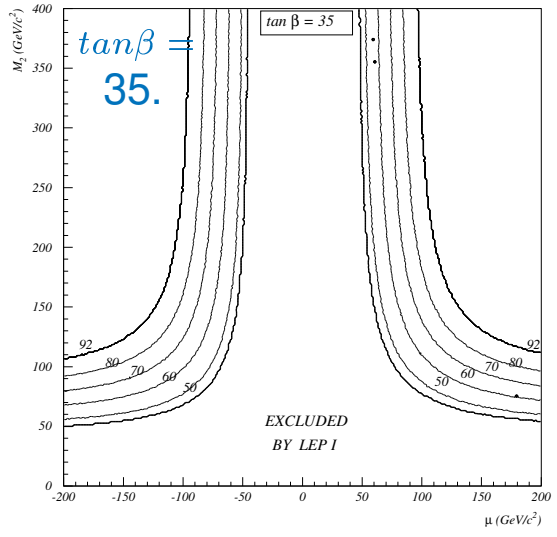


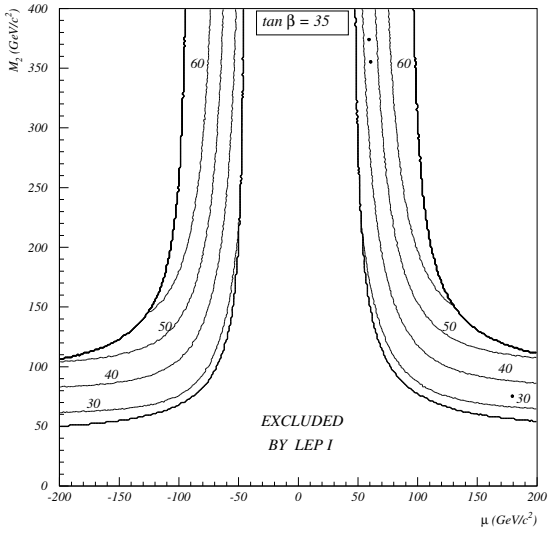
$\tilde{\chi}_1^{0}$











neutralinos: mixings $\tilde{B}, \tilde{W}^3, \tilde{H}_u^0, \tilde{H}_d^0 \to$ 4 eigenstates $\tilde{\chi}^0_{1,\ldots,4}$

$$\mathcal{L}_0 = -\frac{1}{2}\psi_0^T \mathcal{M}_0 \psi_0 + h.c.$$

$$\text{with } \psi_0 = \begin{pmatrix} \tilde{B} \\ \tilde{W}^3 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \end{pmatrix} \qquad \text{[2 components]}$$

$$\mathcal{M}_0 = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{pmatrix}$$

physical states: $\chi^0 = N\psi_0$ with $N^\dagger N = \mathbb{1}$

$$\Longrightarrow N^* \mathcal{M}_0 N^{-1} = \begin{pmatrix} m_{\chi_1^0} & 0 & 0 & 0 \\ 0 & m_{\chi_2^0} & 0 & 0 \\ 0 & 0 & m_{\chi_3^0} & 0 \\ 0 & 0 & 0 & m_{\chi_4^0} \end{pmatrix}$$

$$\mathcal{L}_0 = -\frac{1}{2}\sum_{i=1}^{4} m_{\chi_i^0} \overline{\tilde{\chi}_i^0} \tilde{\chi}_i^0 \qquad \text{with}$$

$$\tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^0 \\ (\chi_i^0)^\dagger \end{pmatrix} \qquad \text{[4 components]}$$

limits: $\mu, M_1, M_2 \gg M_Z$

$$
\begin{aligned}
m_{\chi_1^0} &= M_1 - \frac{M_Z^2 s_W^2 (M_1 + \mu s_\beta)}{\mu^2 - M_1^2} \\
m_{\chi_2^0} &= M_2 - \frac{M_Z^2 c_W^2 (M_1 + \mu s_\beta)}{\mu^2 - M_2^2} \\
m_{\chi_3^0} &= |\mu| + \frac{M_Z^2 (1 - \epsilon s_{2\beta})(|\mu| + M_1 c_W^2 + M_2 s_W^2)}{2(|\mu| + M_1)(|\mu| + M_2)} \\
m_{\chi_4^0} &= |\mu| + \frac{M_Z^2 (1 + \epsilon s_{2\beta})(|\mu| - M_1 c_W^2 - M_2 s_W^2)}{2(|\mu| - M_1)(|\mu| - M_2)}
\end{aligned}
$$

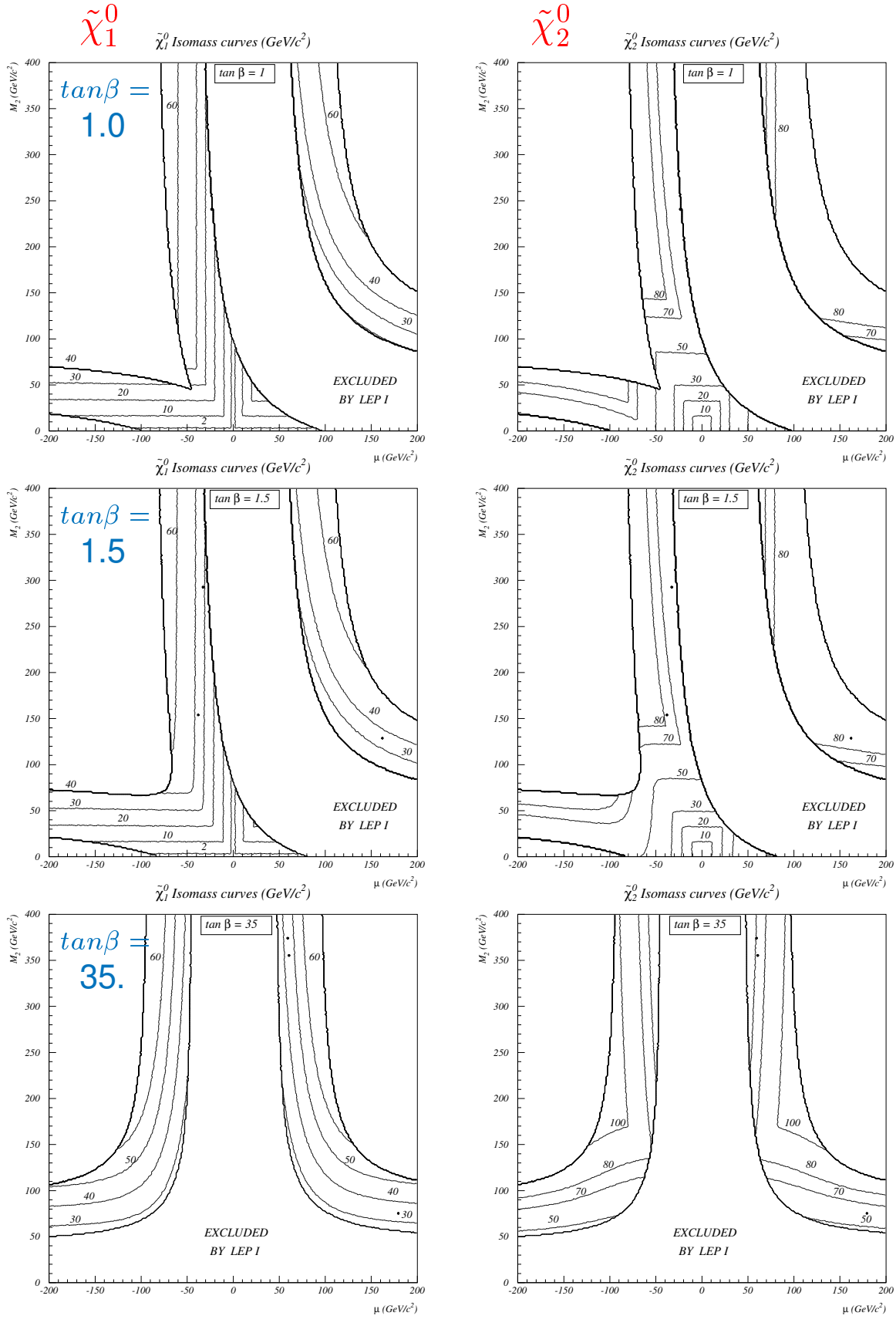
$$
\Rightarrow \quad \tilde{\chi}_1^0 \sim \tilde{B}^0, \tilde{\chi}_2^0 \sim \tilde{W}^3, \tilde{\chi}_{3,4}^0 \sim \frac{\tilde{H}_d^0 \pm \tilde{H}_u^0}{\sqrt{2}}
$$

$$
m_{\tilde{\chi}_2^0} \sim m_{\tilde{\chi}_1^\pm}
$$

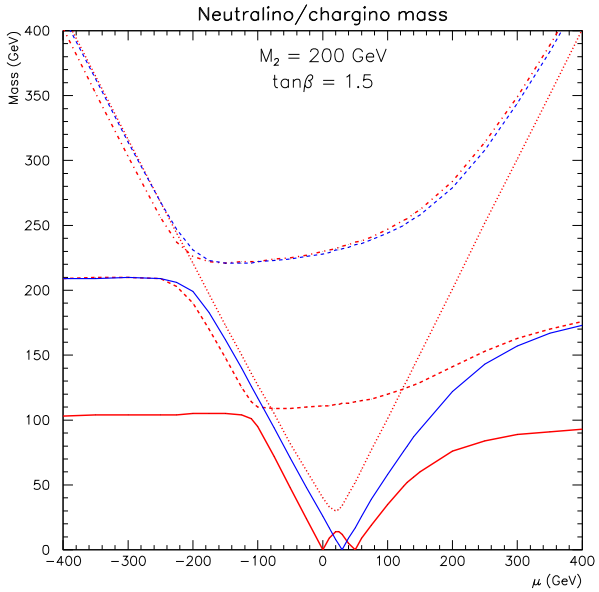
- $|\mu| \gg M_1, M_2$: LSP gaugino–like
- $|\mu| \ll M_1, M_2$: LSP higgsino–like; $m_{\chi_1^0} \sim m_{\chi_2^0} \sim m_{\chi_1^\pm}$ [sorted]

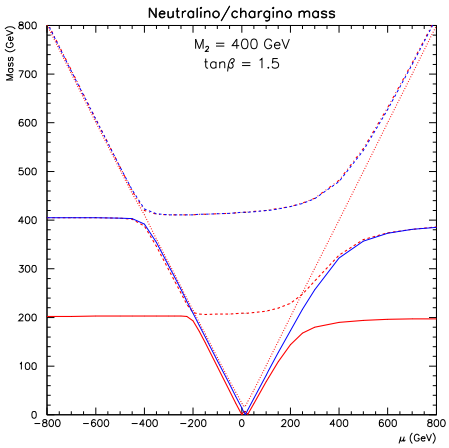
Chargino-Neutralino mass m_2 vs μ

$\tilde{\chi}_1^0$ $\tilde{\chi}_2^0$



Chargino-Neutralino masses, cntd





mass solutions for $M_2 \geq 100GeV$,
pure if $\geq 80\%$, mostly if $\geq 50\%$

$\mu \leq -M_2$:	$\tilde{m}_1 \simeq M_1$	pure Bino
	$\tilde{m}_2 \simeq M_2 \simeq M(\tilde{\chi}_1^{\pm})$	mostly Wino
	$\tilde{m}_3 \simeq \lvert\mu\rvert \simeq M(\tilde{\chi}_2^{\pm})$	pure higgsino a
	$\tilde{m}_4 \simeq \lvert\mu\rvert > \simeq \tilde{m}_3$	pure higgsino b
$-M_2 \leq \mu \leq -M_1$:	$\tilde{m}_1 \simeq M_1$	pure Bino
	$\tilde{m}_2 \simeq \lvert\mu\rvert \simeq < M(\tilde{\chi}_1^{\pm})$	pure higgsino b
	$\tilde{m}_3 \simeq \lvert\mu\rvert > \simeq M(\tilde{\chi}_1^{\pm})$	pure higgsino a
	$\tilde{m}_4 \simeq M_2 \simeq M(\tilde{\chi}_2^{\pm})$	mostly Wino
$-M_1 \leq \mu \leq M_1$:	$\tilde{m}_1 \simeq \lvert\mu\rvert \simeq < M(\tilde{\chi}_1^{\pm})$	pure higgsino
	$\tilde{m}_2 \simeq M_1$	pure Bino
	$\tilde{m}_3 \simeq \lvert\mu\rvert > \simeq M(\tilde{\chi}_1^{\pm})$	pure higgsino
	$\tilde{m}_4 \simeq M_2 \simeq M(\tilde{\chi}_2^{\pm})$	mostly Wino
	$\tilde{m}_1 \simeq < M(\tilde{\chi}_1^{\pm}) \leq \tilde{m}_3 \leq \tilde{m}_2$	
$M_1 \leq \mu \leq M_2$:	$\tilde{m}_1 \simeq < M_1 < M(\tilde{\chi}_1^{\pm})$	mixed Bino/Wino/higgsino
	$\tilde{m}_2 > M(\tilde{\chi}_1^{\pm})$ increases slowly to M_2	mixed Bino/higgsino
	$\tilde{m}_3 \simeq \lvert\mu\rvert$	pure higgsino b
	$\tilde{m}_4 \simeq \lvert\mu\rvert > \simeq \tilde{m}_3 \simeq M(\tilde{\chi}_2^{\pm})$	mostly Wino
$\mu \geq M_2$:	$\tilde{m}_1 \simeq M_1$	mostly photino
	$\tilde{m}_2$ approaches $M_2, \simeq M(\tilde{\chi}_1^{\pm})$	mostly Zino
	$\tilde{m}_3 \simeq \lvert\mu\rvert$	pure higgsino a
	$\tilde{m}_4 \simeq \lvert\mu\rvert \simeq M(\tilde{\chi}_2^{\pm})) \simeq > \tilde{m}_3$	mostly higgsino b

3.) SFERMIONS

$$\mathcal{L}_{\tilde{f}} = -\tilde{f}^{\dagger}\mathcal{M}^2_{\tilde{f}}\tilde{f} \quad \text{with } \tilde{f} = \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} M^2_{LL} & M^2_{LR} \\ M^2_{RL} & M^2_{RR} \end{pmatrix}$$

$$M^2_{LL/RR} = M^2_{\tilde{f}_{L/R}} + m^2_f \underbrace{\pm M^2_Z (I_{3L/R} - e_f s^2_W) c_{2\beta}}_{D \text{ term}}$$

$$M^2_{LR} = M^2_{RL} = m_f \left[A_f - \mu r_f\right]$$

$$r_f = \begin{cases} \cot\beta & \text{for} \quad f = \text{up–type} \\ \text{tg}\beta & \text{for} \quad f = \text{down–type} \end{cases}$$

mixing: large only in 3. generation: $L, R \to 1, 2$

$$m^2_{\tilde{f}_{1/2}} = m^2_f + \frac{1}{2}\left\{M^2_{\tilde{f}_L} + M^2_{\tilde{f}_R} \mp \sqrt{(M^2_{\tilde{f}_L} - M^2_{\tilde{f}_R})^2 + 4m^2_f (A_f - \mu r_f)^2}\right\}$$

[w/o D term]

diagonalisation:

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_f & \sin\theta_f \\ -\sin\theta_f & \cos\theta_f \end{pmatrix} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}$$

$$\sin 2\theta_f = \frac{2m_f(A_f - \mu r_f)}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}}$$

$$\cos 2\theta_f = \frac{M^2_{\tilde{f}_L} - M^2_{\tilde{f}_R}}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}}$$

- splitting largest for heaviest fermions

$\Rightarrow \tilde{t}_1$ can be the lightest squark

$\tilde{\tau}_1$ can be the lightest slepton

- large tgβ: splitting $\tilde{b}_1 \leftrightarrow \tilde{b}_2$ large

$\Rightarrow$ there are regions in which $\tilde{b}_1$ is lightest squark

Example of SUSY Spectrum

Schematic Sparticle Spectrum in MSSM

