§2. SUSY Breaking

attempt: spontaneous symmetry breaking

define: $[H, Q] = 0$, but $Q|0\rangle, Q^\dagger|0\rangle \neq 0$

algebra: $\{Q_\alpha, \bar{Q}_\beta\} = 2\not{P}_{L\alpha\beta}$

$$\Longrightarrow \underline{\underline{H = \frac{1}{4}\sum_\alpha \left[Q_\alpha Q_\alpha^\dagger + Q_\alpha^\dagger Q_\alpha\right]}}$$

$\langle 0|H|0\rangle = \frac{1}{4}\sum_\alpha \left[\| Q_\alpha^\dagger|0\rangle \|^2 + \| Q_\alpha|0\rangle \|^2\right] = 0$ with SUSY

> 0 with ~~SUSY~~

potential: $V = \sum_i |F_i|^2 + \frac{1}{2}\sum_a |D_a|^2 \geq 0$

SUSY exact: $F_i(\Phi_0) = 0$ and $D_a(\Phi_0, \Phi_0^*) = 0$

SUSY broken: $D \neq 0$ D–breaking
(Fayet mechanism)

$F \neq 0$ F–breaking
(O'Raifeartaigh)

a) D–breaking:

chiral/antichiral field with U(1) gauge invariance:

$$\begin{aligned}\mathcal{L} &= \frac{1}{4}\int d^2\Theta\; V_\alpha^2 + h.c. \\ &+ \int d^4\Theta \left[\Phi_+^* e^{gV}\Phi_+ + \Phi_-^* e^{-gV}\Phi_-\right] \\ &+ m\int d^2\Theta\Phi_+\Phi_- + h.c. \; -2k\int d^4\Theta V\end{aligned}$$

[SUSY and gauge invariant: $V \to V + \underbrace{\chi + \chi^\dagger}_{\text{(anti)chiral}}$]

potential: $\Phi_\pm \sim \phi_\pm + \Theta\psi_\pm + \Theta^2 F_\pm$

$$V \sim \Theta\sigma^\mu\bar{\Theta}A_\mu + i\Theta^2\bar{\Theta}\bar{\lambda} + h.c. + \Theta^2\bar{\Theta}^2 D$$

$$\begin{aligned}V_{pot} &= -\left[\frac{1}{2}D^2 + |F_+|^2 + |F_-|^2\right] - kD \\ &- \frac{g}{2}D\left[\phi_+^*\phi_+ - \phi_-^*\phi_-\right] \\ &- m\left[F_+^*\phi_+^* + F_-^*\phi_-^* + F_+\phi_+ + F_-\phi_-\right]\end{aligned}$$

Lagrange: $D + \frac{g}{2}\left[\phi_+^*\phi_+ - \phi_-^*\phi_-\right] + k = 0$

$$F_\pm + m\phi_\pm^* = 0$$

$$\begin{aligned}V_{pot} &= \frac{1}{2}k^2 + \left(m^2 + \frac{kg}{2}\right)\phi_+^*\phi_+ + \left(m^2 - \frac{kg}{2}\right)\phi_-^*\phi_- \\ &+ \frac{g^2}{8}\left(\phi_+^*\phi_+ - \phi_-^*\phi_-\right)^2 \\ &\geq 0 \qquad [\longrightarrow \langle 0|H|0\rangle \geq 0]\end{aligned}$$

spontaneous symmetry breaking:
$\langle F_\pm \rangle = 0$
$\langle D \rangle = -k \neq 0$: D–breaking

masses: $\boxed{\begin{array}{rcl} m(\psi^\pm) & = & m \\ \\ m(\phi^\pm) & = & \sqrt{m^2 \pm \frac{1}{2}kg} \end{array}}$

b) F–breaking:

superfield S: action $= \int d^4\Theta S^\dagger S + f \int d^2\Theta S + h.c.$
$\Rightarrow W(S) = fS$
$\rightarrow F_S = -\left(\frac{\partial}{\partial\phi} f\phi\right)^* = -f^* \neq 0$: F–breaking
$\rightarrow \underline{\underline{\mathcal{L} = |\partial_\mu \phi|^2 + \bar{\psi}_L i\partial\!\!\!/ \psi_L - |f|^2}}$

- massless scalar + fermion
- plus constant energy density
  $V = |f|^2 \leftarrow$ relevant with further coupling
  $\longrightarrow$ mass splitting in SUSY multiplets

FERRARA–GIRARDELLO–PALUMBO–mass formula

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{kin} + \mathcal{L}_Y - V : \\
\mathcal{L}_Y &= -\frac{1}{2}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\overline{\psi_{iL}{}^c}\psi_{jL} - i\sqrt{2}\bar{\lambda}^a\psi_{iL}T^a_{ij}\phi^*_j + h.c. \\
V &= \left|\frac{\partial W}{\partial\phi_i}\right|^2 + \frac{1}{2}\left(\phi^*_i T^a_{ij}\phi_j\right)^2 \qquad [g=1]
\end{aligned}$$

1.) boson mass matrix:

$$\begin{aligned}
\mathcal{L}_{M_0} &= \phi_i (M_0^2)_{ij^*}\phi^*_j \longrightarrow (M_0^2)_{ij^*} = \frac{\partial^2 V}{\partial\phi_i\partial\phi^*_j} \\
M^2_{0ij^*} &= W''_{ik}W^{*\prime\prime}_{kj^*} + T^a_{j^*i}(\phi^* T^a\phi) + (T^a\phi)_{j^*}(\phi^* T^a)_i
\end{aligned}$$

2.) fermion mass matrix:

$$\sqrt{2}M_{1/2} = \begin{matrix} \\ \psi_L \\ \lambda \end{matrix}\begin{matrix} \begin{matrix} \psi_L & \lambda \end{matrix} \\ \begin{pmatrix} W''_{ij} & \sqrt{2}\phi^* T^a \\ \sqrt{2}T^a\phi & 0 \end{pmatrix} \end{matrix}$$

3.) gauge boson mass matrix:

$$M^2_{1ab} = \left(\phi^* T^a, T^b\phi\right) \qquad \left[|D_\mu\phi|^2 \to (\phi^* g V^\mu)(g V_\mu\phi)\right]$$

mass sum rule:

$$Tr\left[M_0^2 - 2M_{1/2}^2 + 3M_1^2\right] = 0$$

$$\boxed{\sum_J (-1)^{2J}(2J+1)TrM_J^2 = 0}$$

valid with spontaneous symmetry breaking [F–breaking and D–breaking with anomaly freedom]
$\longrightarrow$ at least one boson mass must be smaller than the corresponding fermion mass: experimental contradiction

possible solution: RHS $\neq 0$ via supergravitaty sector $\sim m_{3/2}^2$
$\longrightarrow$ soft, i.e. renormalizable, SUSY breaking in Standard-Model sector [via explicit mass terms]

SOFT SUSY BREAKING

Definition: SUSY breaking operators such that no new quadratic divergences are generated

Realisation:
- spont. SUSY breaking in "hidden sector"
- mediated into eigenworld via gravity etc.
- apparent explicit SUSY breaking in eigenworld

Execution:
— gauge and gauge-related coupl. unchanged: $\not\Lambda^2$
— SUSY masses $\neq$ SM masses
— scalar couplings in superpotential modified

Lagrange density:

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}[SUSY] + \mathcal{L}[\cancel{SUSY}] \\
\mathcal{L}[\cancel{SUSY}] &= -m^2_{\tilde{f}}|\tilde{f}|^2 + \cdots && \text{sfermions, Higgs} \\
&- W_2(\phi) - W_3(\phi) && \text{superpotential} \\
&- \frac{1}{2} M_i \bar{\lambda}_i \lambda_i && \text{gauginos}
\end{aligned}$$