II GUT

§1. Unification of Gauge Couplings

$SU(3) \times SU(2) \times U(1) \to 3$ gauge couplings
$\to \sin^2\theta_W$ free parameter

$G \supset SU(3) \times SU(2) \times U(1) \to 1$ gauge coupling
$\to \sin^2\theta_W$ fixed

$$\begin{aligned}\mathcal{L}_{int} &= -g\bar{\psi}\gamma^\mu \left[W_\mu^3 I_3 + B_\mu T_Y + \cdots\right]\psi \qquad (T_Y = \kappa\frac{Y}{2}) \\ &= -\underbrace{g\sin\theta_W}_{e}\,\bar{\psi}\gamma^\mu\,\underbrace{(I_3 + T_Y\cot\theta_W)}_{\text{charge } Q} \\ &\qquad \underbrace{\left(B_\mu\cos\theta_W + W_\mu^3\sin\theta_W\right)}_{A_\mu}\psi + \cdots\end{aligned}$$

summation over multiplets [SU(5)]:

$$\sum Q^2 = \sum I_3^2 + 2\underbrace{\sum I_3 T_Y}_{=0}\cot\theta_W + \cot^2\theta_W\underbrace{\sum T_Y^2}_{=\sum I_3^2}$$

$$\Longrightarrow \boxed{\sin^2\theta_W^{sym} = \frac{\sum I_3^2}{\sum Q^2}}$$ Georgi Quinn Weinberg $\qquad \left[\kappa = \tan\theta_W^{sym}\right]$

15 fermions in one generation:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad e_L^+ \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{matrix} u_L^c \\ d_L^c \end{matrix}$$

FG:	2	1	6	6

side note: $\psi_L^c = C\overline{(\psi_R)}^T$

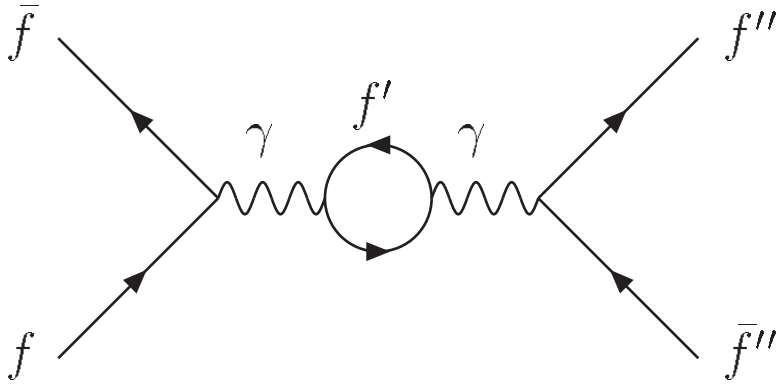
$C = i\gamma^0\gamma^2$

(charge conjugation)

$$\sin^2\theta_W^{sym} = \frac{(2+3\cdot 2)\left(\frac{1}{2}\right)^2}{2\left[(-1)^2+3\left(\frac{2}{3}\right)^2+3\left(-\frac{1}{3}\right)^2\right]} = \frac{3}{8} = 0.375$$

value at the symmetry scale of G

RUNNING COUPLINGS



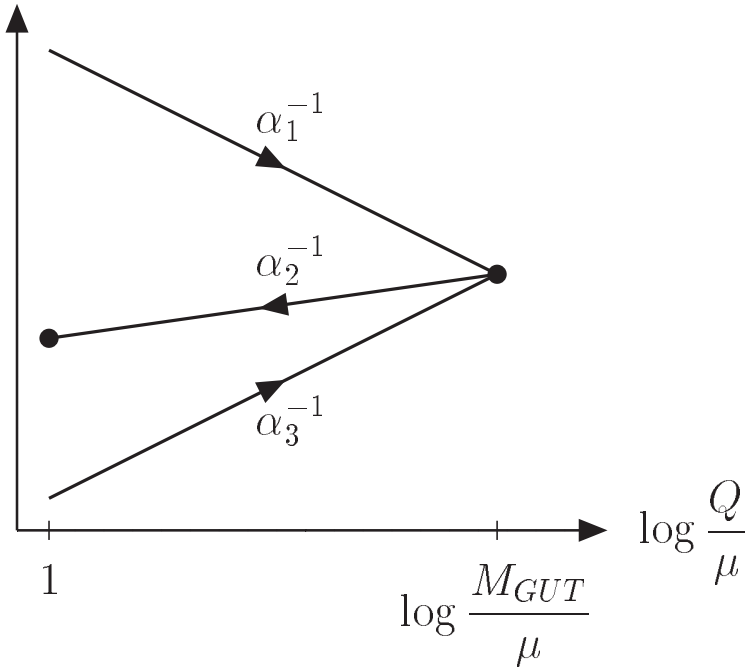
renormalization group equations:

$$\frac{\partial \alpha_i}{\partial \log \mu^2} = -\frac{b_i}{4\pi}\alpha_i^2 + \mathcal{O}(\alpha_i^3)$$
$$\Longrightarrow \frac{1}{\alpha_i(Q^2)} = \frac{1}{\alpha_i(\mu^2)} + \frac{b_i}{4\pi}\log\frac{Q^2}{\mu^2}$$

SM: $\alpha_1 = \frac{5}{3}\frac{g'^2}{4\pi} \quad \alpha_2 = \frac{g^2}{4\pi} \quad \alpha_3 = \frac{g_s^2}{4\pi}$

$$4\pi b_1 = -\frac{4}{3}N_G - \frac{N_H}{10} \qquad [N_G = 3, N_H = 1]$$
$$4\pi b_2 = \frac{22}{3} - \frac{4}{3}N_G - \frac{N_H}{6}$$
$$4\pi b_3 = 11 - \frac{4}{3}N_G$$

$\uparrow$ asympt. freedom



Solution:

$$\log \frac{M_{GUT}}{\mu} = \frac{2\pi}{11\left(1+\frac{N_H}{66}\right)}\left[\frac{1}{2\alpha(\mu^2)} - \frac{4}{3}\frac{1}{\alpha_s(\mu^2)}\right]$$

$$\frac{1}{\alpha_0} = \frac{1}{\alpha_s(\mu^2)} + \frac{b_3}{2\pi}\log\frac{M_{GUT}}{\mu}$$

$$\sin^2\theta_W(\mu^2) = \frac{1}{6}\,\frac{1+\frac{N_H}{22}}{1+\frac{N_H}{66}} + \frac{5}{9}\,\frac{1-\frac{N_H}{110}}{1+\frac{N_H}{66}}\,\frac{\alpha(\mu^2)}{\alpha_s(\mu^2)}$$

$\mu = M_Z = 91.187$ GeV: $\alpha(\mu^2) = \frac{1}{127.9}$

$$\alpha_s(\mu^2) = 0.115\ldots0.123$$

$$\Longrightarrow M_{GUT} = (5.7\ldots8.7)\cdot 10^{14}\text{GeV}$$

$$\alpha_0 \approx 0.024 \approx \frac{1}{40}$$

$$\sin^2\theta_W(\mu^2) = 0.206\ldots0.209$$

[theor. predicted — later exp. "confirmed"]

§2. SU(5)

$SU(5) \supset SU(3) \times SU(2) \times U(1)$ minimal group
24 generators $T^a \Longrightarrow$ 24 gauge bosons

$$\text{color:} \quad T^a = \begin{pmatrix} \frac{\lambda^a}{2} & 0 \\ 0 & 0 \end{pmatrix} \qquad (a = 1, \ldots, 8)$$

$$SU(2)_L: \quad I_1 = T^{22} \quad I_2 = T^{23}$$

$$I_3 = \tfrac{1}{4}(\sqrt{10}T^{24} - \sqrt{6}T^{15})$$

$$I_i \sim \begin{pmatrix} 0 & 0 \\ 0 & \frac{\sigma^i}{2} \end{pmatrix} \qquad (i = 1, \ldots, 3)$$

charge matrix: $Q = 4\sqrt{\frac{1}{6}}\, T^{15}$

hypercharge: [$I_3^c = 4I_2I_3I_2 = -I_3 \leftarrow$ charge conj.]
$Y = \frac{1}{2}\sqrt{10}T^{24} + \frac{5}{6}\sqrt{6}T^{15} = 2(Q - I_3^c)$

FERMIONS

$$5^* = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ \text{- - -} \\ e^- \\ -\nu_e \end{pmatrix}_L$$

color–antitriplet
isosinglet

color–singlet
isodoublet

SU(5) matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^9 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^{10} = \begin{pmatrix} 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^{11} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^{12} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^{13} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^{14} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 \\ 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^{16} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^{17} = \begin{pmatrix} 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda^{18} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^{19} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 & 0 \end{pmatrix} \quad \lambda^{20} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^{21} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 \end{pmatrix} \quad \lambda^{22} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^{23} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & i & 0 \end{pmatrix} \quad \lambda^{24} = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -4 \end{pmatrix}$$

$$T^a = \frac{\lambda^a}{2}$$

$$10 \;=\; \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ \hline u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{array} \right)_L$$

sum of two irreducible representations: no fundamental theory $\rightarrow$ intermediate theory

charge quantization:
$\overline{5^*}\mathcal{Q}5^* = Tr\mathcal{Q} = 0 = 3Q(\bar{d}) + Q(e^-)$

$$\Longrightarrow \boxed{Q(\bar{d}) = -\tfrac{1}{3}Q(e^-) = \tfrac{1}{3}}$$

natural explanation of equality $Q(P) = -Q(e^-)$ [$1 : 10^{21}$]

GAUGE BOSONS

$$V_\mu = V_\mu^a T^a = \frac{1}{\sqrt{2}}*$$

$$\left(\begin{array}{ccc|cc} \frac{G^3}{\sqrt{2}} + \frac{G^8}{\sqrt{6}} - \frac{2B}{\sqrt{30}} & \frac{G^{1-i2}}{\sqrt{2}} & \frac{G^{4-i5}}{\sqrt{2}} & X_1^\dagger & Y_1^\dagger \\ \frac{G^{1+i2}}{\sqrt{2}} & -\frac{G^3}{\sqrt{2}} + \frac{G^8}{\sqrt{6}} - \frac{2B}{\sqrt{30}} & \frac{G^{6-i7}}{\sqrt{2}} & X_2^\dagger & Y_2^\dagger \\ \frac{G^{4+i5}}{\sqrt{2}} & \frac{G^{6+i7}}{\sqrt{2}} & -\sqrt{\frac{2}{3}}G^8 - \frac{2B}{\sqrt{30}} & X_3^\dagger & Y_3^\dagger \\ \hline X_1 & X_2 & X_3 & \frac{W^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & -\frac{W^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} \end{array} \right)$$

Lagrange density: $\mathcal{L}_0 = \overline{5^*} i \not{D} 5^* + Tr\overline{10} i \not{D} 10$

$$iD_\mu = i\partial_\mu - g_5 V_\mu$$

$$\mathcal{L}_{int} = -g_5(\bar{u} \not{G} u + \bar{d} \not{G} d) \quad \textbf{QCD}$$

$$-g_5(\bar{L}_q \not{W} L_q + \bar{L}_e \not{W} L_e) - g_5\sqrt{\frac{3}{5}}\bar{F} \not{B} \frac{Y}{2} F \quad \textbf{GSW}$$

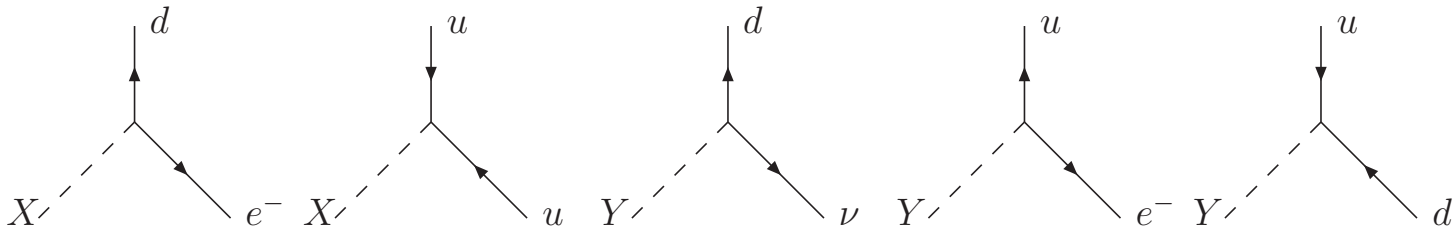
$$-\frac{g_5}{\sqrt{2}}(\bar{d} \not{X} e^c + \epsilon_{ijk}\overline{u^c{}_L}^i \not{X}^j u_L^k + h.c.)$$

L-Q coupling

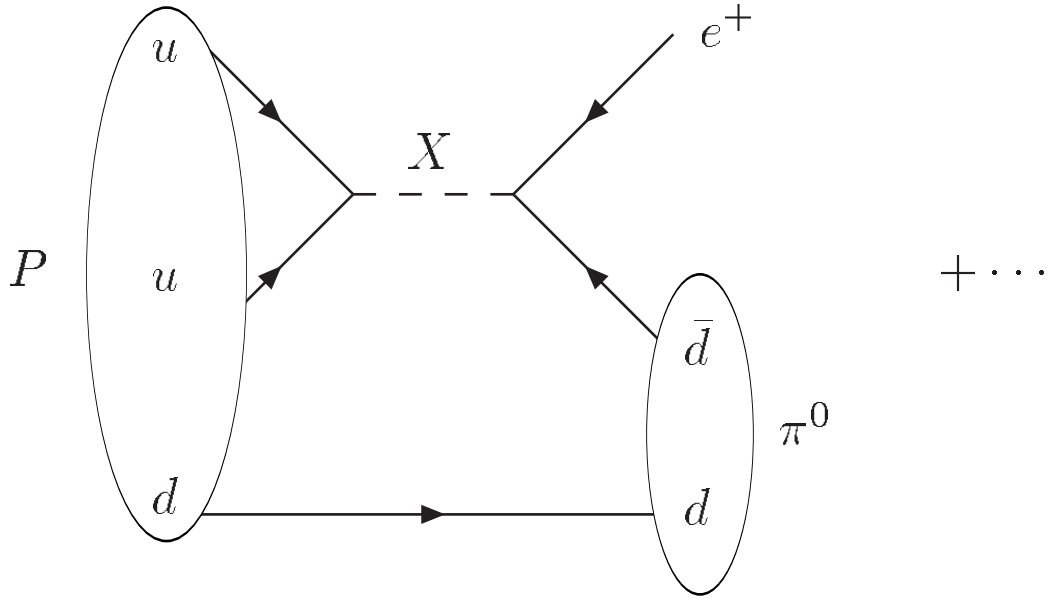
$$-\frac{g_5}{\sqrt{2}}(\bar{d}_R \not{Y} \nu_R^c + \bar{u}_L \not{Y} e_L^c - \epsilon_{ijk}\overline{u^c{}_L}^i \not{Y}^j d_L^k + h.c.)$$

Q-Q coupling

X, Y bosons possess leptoquark und diquark properties



§3. Proton Decay



effective Lagrange density: $\frac{G_*}{\sqrt{2}} = \frac{g_5^2}{8M_{X,Y}^2} \sim \frac{g_5^2}{8M_{GUT}^2}$

$$\begin{aligned}\mathcal{L}_{eff} &= \frac{4G_*}{\sqrt{2}}\epsilon_{ijk}\Big[\overline{{u^c}_L}^k\gamma_\mu u_L^j(\overline{{e^c}_L}\gamma^\mu d_L^i + \overline{{e^c}_R}\gamma^\mu d_R^i) \\ &\quad + \overline{{u^c}_L}^j\gamma_\mu d_L^k(\overline{{\nu^c}_R}\gamma^\mu d_R^i + \overline{{e^c}_L}\gamma^\mu u_L^i)\Big] + h.c.\end{aligned}$$

selection rule: $B - L = const \Longrightarrow \boxed{\Delta B = \Delta L}$

dominant decays: $p \to \pi^0 e^+, \rho^0 e^+, \omega e^+, \pi^+ \bar{\nu}_e, \ldots$

$\tau_P(P \to \pi^0 e^+) \sim \left(\frac{M_{X,Y}}{2{\cdot}10^{14}\text{GeV}}\right)^4 * 10^{28\pm1}\ a$

exp.: $\tau_P(P \to \pi^0 e^+) > 2.4 * 10^{34} a \Longrightarrow$ SU(5) excluded [Superkamiokande]

§4. Spontaneous Symmetry Breaking

$$SU(5) \underset{M_{GUT}}{\rightarrow} SU(3)_C \times SU(2)_L \times U(1)_Y \underset{G_F^{-1/2}}{\rightarrow} SU(3)_C \times U(1)_{elm}$$

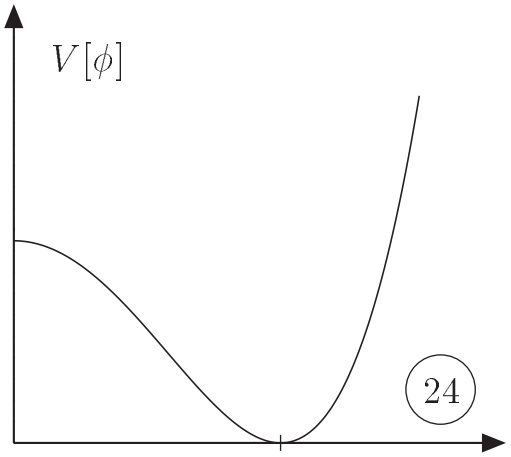
X, Y massive — fermions, W, Z massive

$\underline{24}$ — $\underline{5}$ ←Higgs fields

$$\phi = \phi^a T^a \qquad H = \begin{pmatrix} H_1 \\ H_2 \\ H_3 \\ - \ - \ - \\ H_4 \\ H_5 \end{pmatrix} \begin{matrix} \\ \\ \\ \\ \text{Color–3, } SU_2\text{–1} \\ \\ \text{Color–1, } SU_2\text{–2} \end{matrix}$$

1.) GUT Symmetry Breaking

$$\begin{aligned} \mathcal{L}_\phi &= Tr|D_\mu\phi|^2 - M_{24}^2 Tr|\phi|^2 - \lambda_{24}\left(Tr|\phi|^2\right)^2 \\ &- \lambda'_{24} Tr|\phi|^4 \\ iD_\mu\phi &= i\partial_\mu\phi - g_5\,[V_\mu, \phi] \end{aligned}$$



for $M_{24}^2 < 0, \lambda'_{24} > 0, \lambda_{24} > -\frac{7}{30}\lambda'_{24}$

$$\phi_0 = \frac{v_{24}}{\sqrt{15}} \left(\begin{array}{ccc|cc} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ \hline & & & -\frac{3}{2} & \\ & & & & -\frac{3}{2} \end{array} \right)$$

$$v_{24}^2 = -\frac{M_{24}^2}{\lambda_{24} + \frac{7}{30}\lambda'_{24}}$$

minimum: $SU_3 \times SU_2 \times U_1$ symmetric

X, Y masses: $\mathcal{L}^M_{X,Y} = -g_5^2 Tr|[V_\mu, \phi_0]|^2$

$$\Longrightarrow \boxed{M_X^2 = M_Y^2 = \frac{5}{12} g_5^2 v_{24}^2}$$

P decay $\Longrightarrow v_{24} \sim 10^{16}$ GeV $\sim 10^{-3} M_{Planck}$

2.) Symmetry Breaking in SM

$$\mathcal{L}_H = -\frac{1}{2}M_5^2 H^\dagger H - \frac{\lambda_5}{2}(H^\dagger H)^2 = -V(H)$$

spontaneous symmetry breaking:

$$\left.\begin{array}{r} M_5^2 < 0 \\ \lambda_5 > 0 \end{array}\right\} \Longrightarrow H_{50} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_5 \end{pmatrix} \quad \begin{array}{l} v_5^2 = -\dfrac{M_5^2}{\lambda_5} \\ \\ v_5 = \dfrac{v}{\sqrt{2}} \end{array}$$

$$\left.\begin{array}{l} \text{gluons massless} \\ W, Z \text{ massive} \\ \gamma \text{ massless} \end{array}\right\} SU_3 \times SU_2 \times U_1 \underset{v}{\rightarrow} SU_3 \times U_{1elm}$$

$$\begin{aligned} \mathcal{L}_{Yuk} &= -G_1 \overline{5_L^{*c}} 10 H^* - G_2 \epsilon_{ijklm} \left(\overline{10_L^c}\right)_{ij} (10_L)_{kl} H_m \\ &= -\frac{G_1 v_5}{\sqrt{2}} \left(\bar{d}d + \bar{e}e\right) + \cdots \end{aligned}$$

mass relations at symmetry point:

$$\boxed{\begin{array}{rcl} m_\nu &=& 0 \\ m_u &=& 4G_2 v_5 \\ m_d = m_e &=& G_1 v_5/\sqrt{2} \end{array}} \qquad \boxed{\begin{array}{c} m_d = m_e \\ m_s = m_\mu \\ \boxed{m_b = m_\tau} \end{array}}$$

RG evolution: $\boxed{\dfrac{m_b(\sim \text{GeV})}{m_\tau(\sim \text{GeV})} = 3.1 \ldots 3.4}$ $\sim$ o.k.

§5. $SU(5)$ Assessment

+	−
$SU(5) \supset SU(3) \times SU(2) \times U(1)$	family problem: P decay
1 coupling	many parameters
$\sin^2 \theta_W \approx 0.20$	$\mathcal{CP}$, quark mixing
charge quantization	multiplets added
$m_b/m_\tau \approx 3$	hierarchy problem
"massless" ν's	gravity

other groups:
(i) $SO(10)$: rank 5, 45 gauge bosons

$$SO(10) \longrightarrow SU(5) \times U(1) \longrightarrow SU(3) \times SU(2) \times U(1)$$

or $\longrightarrow SU(4) \times SU(2)_L \times SU(2)_R$

$SU(4) \supset SU(3)_C \times U(1)_\ell$
lepton extension of QCD [Pati–Salam]

$$SU(2)_L \times SU(2)_R \underset{M_1}{\rightarrow} SU(2)_L \times U(1)_Y \underset{\sim M_W}{\rightarrow} U(1)_{elm}$$

$[\sim 10^{10}$ GeV$]$

$M_{W_R} \gg M_{W_L}$: parity violation spontaneous
fermions: 16–dim. fundamental representation

$$\longrightarrow \nu_R \longrightarrow \nu \text{ oscillations}$$

(ii) E_6: rank 6, 78 gauge bosons

$E_6 \quad \rightarrow SO(10) \rightarrow \cdots$ [motivated by string theories]

or $\rightarrow SU(3)_C \times SU(3)_L \times SU(3)_R$
$SU(3)_L \longrightarrow SU(2)_L \times U(1)_L$
$\Longrightarrow Z'$ boson with $M_{Z'} \leq \mathcal{O}(1\text{TeV})$

fermions: 27–dim. fundamental representation
$\Longrightarrow$ heavy leptons, quarks [Iso–1, Iso–2]

Z', exotic fermions: good exp. questions for TeV colliders