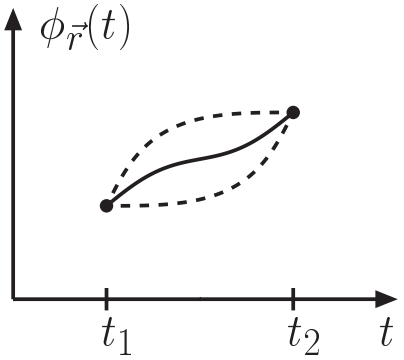
I STANDARD MODEL

§1. Lagrange Formalism

Classical mechanics	Field theory
d.o.f.: coordinates $q(t)$	field amplitudes $\phi(t,\vec{r}) = \phi(x)$
Action: $S = \int_{t_1}^{t_2} dt\ L(q,\dot{q})$	$S = \int_{t_1}^{t_2} dt\ \int d^3\vec{r}\mathcal{L}\left[\phi(x), \partial_\mu\phi(x)\right]$
Lagrange function	$= \int d^4x\ \mathcal{L}(\phi, \partial_\mu\phi)$
	Lagrange density

Fields develop with time such that action is minimalized:



$$\delta S = \int d^4x \left\{ \frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\delta\partial_\mu\phi \right\}$$

$$= \int d^4x \left\{ \frac{\partial\mathcal{L}}{\partial\phi}\,\delta\phi + \underbrace{\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\delta\phi\right)}_{=0 \text{ on surface}} - \left(\partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\right)\delta\phi \right\}$$

$$= \int d^4x \left\{ \frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi} \right\}\delta\phi = 0$$

Euler–Lagrange equations: $\boxed{\partial_\mu\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi} - \frac{\partial\mathcal{L}}{\partial\phi} = 0}$

Examples:

1.) Klein–Gordon field [spin 0]

$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 |\phi|^2$$

$$\longrightarrow \boxed{\left(\partial^2 + m^2\right)\phi = 0} \quad \text{[Klein–Gordon eq.]}$$

2.) Dirac field [spin $\frac{1}{2}$]

$$\mathcal{L} = \bar{\psi}(i\,\partial\!\!\!/ - m)\psi$$

$$\longrightarrow \boxed{\left(i\,\partial\!\!\!/ - m\right)\psi = 0} \quad \text{[Dirac equation]}$$

$$\bar{\psi} = \psi^\dagger \gamma^0 \qquad \psi_{L/R} = \frac{1 \mp \gamma_5}{2}\psi$$

$$a\!\!\!/ = a_\mu \gamma^\mu$$

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

$$\gamma_5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

Clifford algebra:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1} \qquad \{\gamma^\mu, \gamma_5\} = 0$$

3.) Maxwell field [spin 1]

Vector potential $A^\mu = (\phi, \vec{A})$:
$\vec{E} = -\text{grad}\phi - \partial_t \vec{A},\ \vec{B} = \text{rot}\vec{A}$

Field strength tensor:

$$F^{\mu\nu} = \partial^\nu A^\mu - \partial^\mu A^\nu = \begin{pmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & B^3 & -B^2 \\ -E^2 & -B^3 & 0 & B^1 \\ -E^3 & B^2 & -B^1 & 0 \end{pmatrix}_{\mu\nu}$$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$\longrightarrow \boxed{\partial^2 A^\mu - \partial^\mu \partial_\nu A^\nu = 0} \quad \text{[Maxwell eq.]}$$

§2. Gauge Theories

1. Abelian Gauge Theories [QED]

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

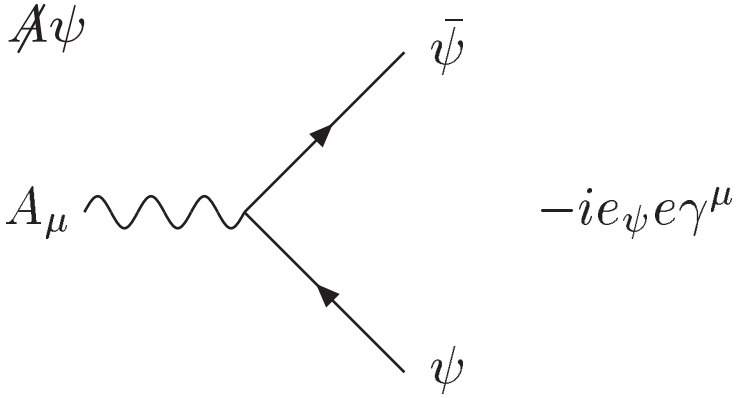
minimal substitution: $i\partial_\mu \to iD_\mu = i\partial_\mu - eA_\mu\mathcal{Q}$

[$\mathcal{Q}$ = charge operator: $\mathcal{Q}\psi = e_\psi\psi$]

invariant under local gauge transformations:

$$\begin{aligned} \psi &\to e^{-i\alpha(x)\mathcal{Q}}\psi \qquad \bar{\psi} \to e^{i\alpha(x)\mathcal{Q}}\bar{\psi} \\ A_\mu &\to A_\mu + \frac{1}{e}\partial_\mu\alpha \quad [F_{\mu\nu} \to F_{\mu\nu}] \end{aligned}$$

$$\mathcal{L}_{int} = -e_\psi e\bar{\psi}\not{A}\psi$$



2. Non–Abelian Gauge Theories [e.g. QCD]

Lie group: $S = e^{-i\alpha^a T^a}$

generators: $\boxed{[T^a, T^b] = i f_{abc} T^c}$ [Lie algebra]

$Tr(T^a T^b) = \frac{1}{2}\delta_{ab}$ $T^{a\dagger} = T^a$

SU(2): $T^a = \dfrac{\sigma^a}{2}$ Pauli-matrices

SU(3): $T^a = \dfrac{\lambda^a}{2}$ Gell–Mann-matrices

vector potential: $V_\mu = V_\mu^a T^a$

minimal substitution: $i\partial_\mu \to iD_\mu = i\partial_\mu - gV_\mu$

field strength tensor:
$V_{\mu\nu} = \dfrac{i}{g}[D_\mu, D_\nu] = \partial_\nu V_\mu - \partial_\mu V_\nu - ig[V_\mu, V_\nu]$

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2} Tr\,(V^{\mu\nu} V_{\mu\nu}) + \bar{\psi}(i\not{D} - m)\psi \\ &= -\frac{1}{4} V^{a\mu\nu} V^a_{\mu\nu} + \bar{\psi}_j(i\not{\partial} - m)\psi_j - g\bar{\psi}_j \not{V}^a T^a_{jk} \psi_k\end{aligned}$$

invariant under local gauge transformations

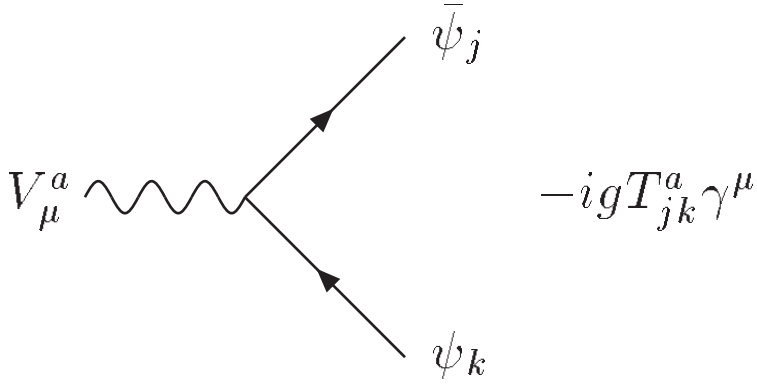
$$\psi \;\to\; S\psi = e^{-i\alpha^a(x) T^a}\psi$$

$$\bar{\psi} \;\to\; \bar{\psi}S^{-1} = \bar{\psi}e^{i\alpha^a(x)T^a} \qquad [S^\dagger = S^{-1}]$$

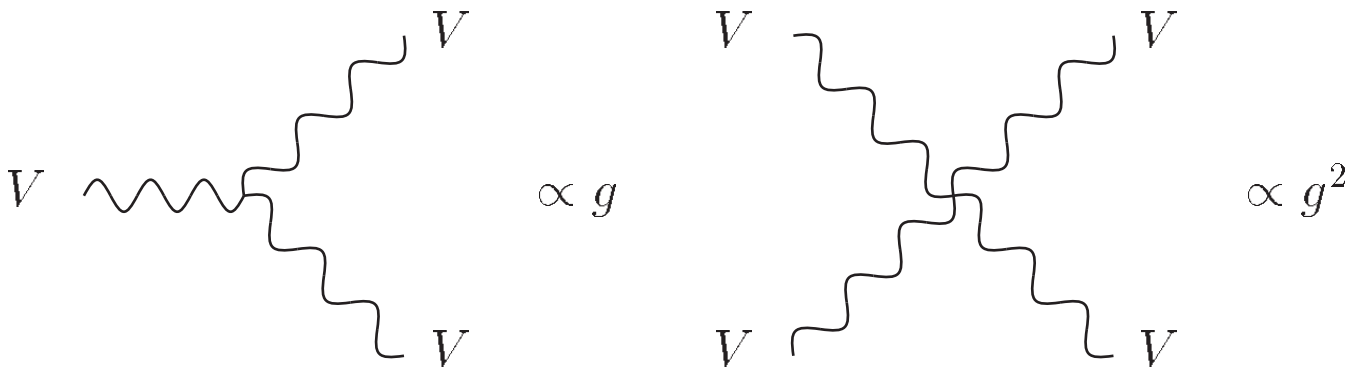
$$V_\mu \;\to\; SV_\mu S^{-1} + \frac{i}{g}(\partial_\mu S)S^{-1}$$

$$\Longrightarrow V_{\mu\nu} \;\to\; SV_{\mu\nu}S^{-1}$$

$$D_\mu \;\to\; SD_\mu S^{-1}$$



self-interactions:



§3. Higgs Mechanism and Standard Model

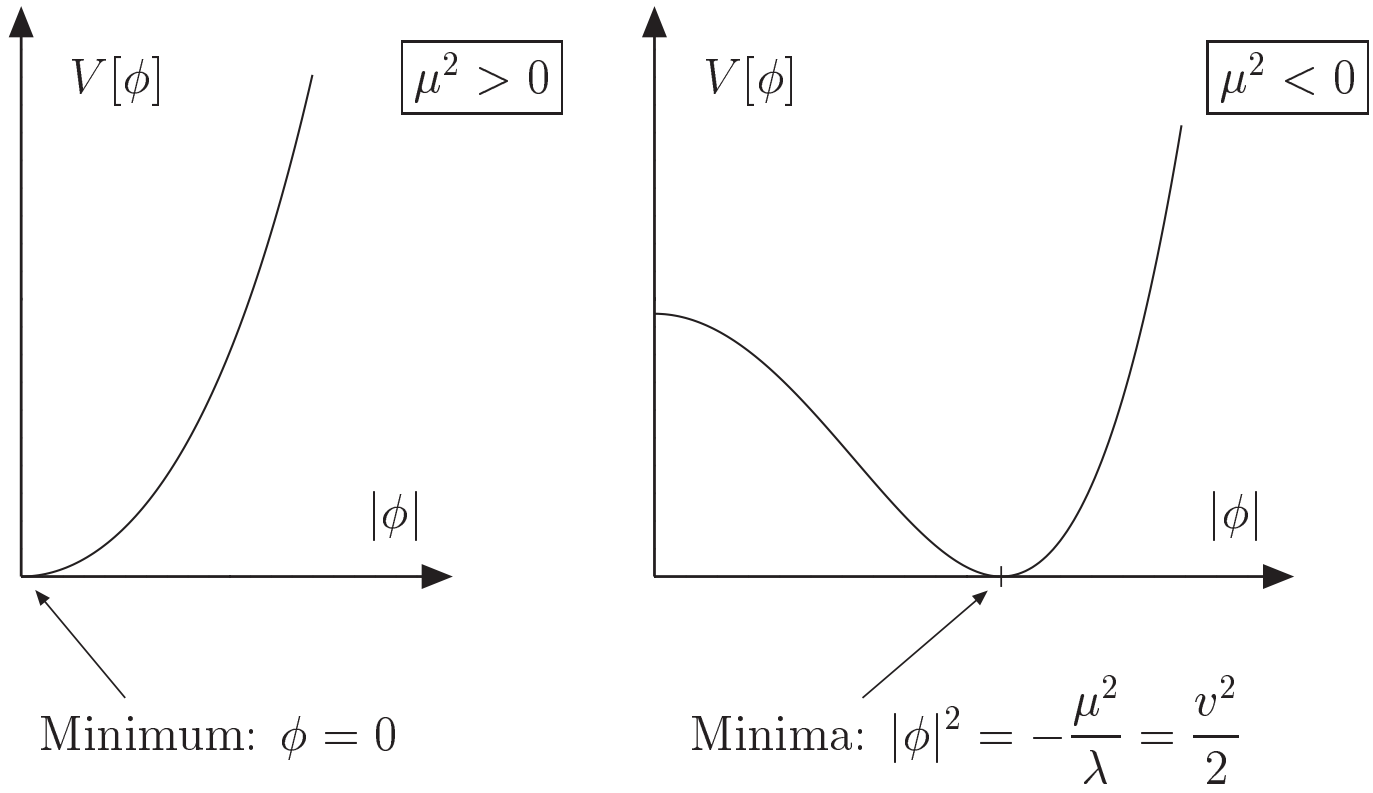
$\mathcal{L}_M = M_V^2 Tr(V^\mu V_\mu)$ destroys gauge invariance
$\rightarrow M_V = 0$?

scalar isospin doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

[isospin $I = \frac{1}{2}$, hypercharge $Y = +1$]

potential: $V(\phi) = \mu^2 \phi^\dagger \phi + \frac{\lambda}{2}(\phi^\dagger \phi)^2$



$\Longrightarrow$ spontaneous symmetry breaking

Higgs field:

$$\phi = \underbrace{e^{-i\frac{\theta^a \sigma^a}{v}}}_{\substack{SU(2)\text{–Trafo} \\ \longrightarrow \text{ gauge trafo}}} \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \Theta^2 - i\Theta^1 \\ v + H + i\Theta^3 \end{pmatrix} + \cdots$$

$\Longrightarrow$ 3 components can be absorbed in $SU(2)$ gauge fields [longitudinal d.o.f.].

fermions:

	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$	e^-_R	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$	μ^-_R	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$	τ^-_R
	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{matrix} u_R \\ d_R \end{matrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{matrix} c_R \\ s_R \end{matrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	$\begin{matrix} t_R \\ b_R \end{matrix}$
isospin:	$\pm\frac{1}{2}$	0	$\pm\frac{1}{2}$	0	$\pm\frac{1}{2}$	0

hypercharge: $Y = 2(Q - I_3)$
[Gell-Mann–Nishijima relation]

gauge bosons: hypercharge $B_\mu \dfrac{Y}{2}$

isospin $W_\mu = W^a_\mu \dfrac{\sigma^a}{2}$

color $G_\mu = G^a_\mu \dfrac{\lambda^a}{2}$

Lagrange density of Standard Model:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ \sum_f \bar{L}_f(i\not{D})L_f + \sum_f \bar{f}_R(i\not{D})f_R \\ &+ |D_\mu\phi|^2 - \frac{\lambda}{2}\left[|\phi|^2 - \frac{v^2}{2}\right]^2 \\ &- \sum_f g_f \bar{L}_f(\phi + \phi^c)f_R + h.c.\end{aligned}$$

charge conjugation: $\phi^c = i\sigma^2\phi^*$

minimal substitution:

$$iD_\mu = i\partial_\mu - g_s G^a_\mu \frac{\lambda^a}{2} - gW^a_\mu \frac{\sigma^a}{2} - g'B_\mu \frac{Y}{2}$$

ground state:

$$\phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad \text{[unitary gauge]}$$

gauge boson masses:

$$\begin{aligned}|D_\mu\phi_0|^2 &= \left|\left(igW^a_\mu\frac{\sigma^a}{2} + ig'B_\mu\frac{Y}{2}\right)\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}\right|^2 \\ &= \frac{v^2}{8}\begin{pmatrix} W^1_\mu \\ W^2_\mu \\ W^3_\mu \\ B_\mu \end{pmatrix}\begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}\begin{pmatrix} W^1_\mu \\ W^2_\mu \\ W^3_\mu \\ B_\mu \end{pmatrix}\end{aligned}$$

$$\Longrightarrow \boxed{\begin{array}{rcl} M_W^2 &=& \dfrac{g^2}{4}v^2 \\ M_Z^2 &=& \dfrac{g^2+g'^2}{4}v^2 \\ M_\gamma^2 &=& 0 \end{array}}$$

$$W_\mu^\pm = \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$[\theta_W = \text{Weinberg angle}]$

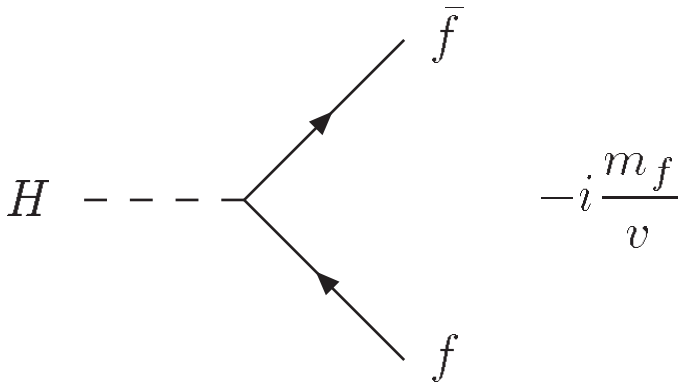
$\longrightarrow$ W und Z bosons acquire mass by Higgs mechanism. Photon remains massless.

Yukawa interaction:

$$\mathcal{L}_{Yuk} = -\sum_f g_f \bar{L}_f(\phi + \phi^c) f_R + h.c. = -\sum_f \frac{g_f}{\sqrt{2}} \bar{f}(v+H) f$$

$$\Longrightarrow \boxed{m_f = \frac{g_f}{\sqrt{2}} v}$$

Higgs mechanism generates fermion masses



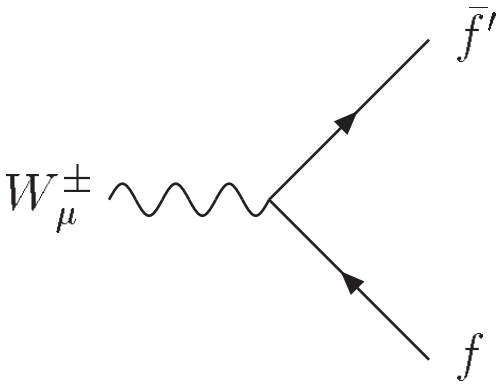
$$-i\frac{m_f}{v}$$

gauge interactions:

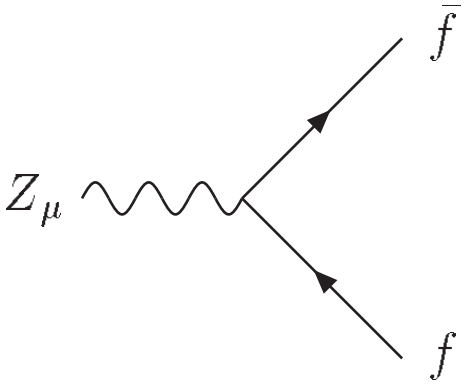
$$\begin{aligned}\mathcal{L}_{int} &= -\frac{g}{\sqrt{2}} j_\mu^+ W^{+\mu} + h.c. \\ &- \frac{g}{\cos\theta_W}\left[j_\mu^3 - \sin^2\theta_W j_\mu^{elm}\right] Z^\mu - e j_\mu^{elm} A^\mu\end{aligned}$$

$$\boxed{e = g\sin\theta_W = g'\cos\theta_W}$$

$$\begin{aligned} j_\mu^\pm &= \bar{L}_f \gamma_\mu \sigma^\pm L_f \qquad\qquad \sigma^\pm = \frac{\sigma^1 \mp i\sigma^2}{2} \\ j_\mu^3 &= \bar{L}_f \gamma_\mu \frac{\sigma^3}{2} L_f \\ j_\mu^{elm} &= e_f \bar{f}\gamma_\mu f = e_f\left\{\bar{f}_L\gamma_\mu f_L + \bar{f}_R\gamma_\mu f_R\right\}\end{aligned}$$



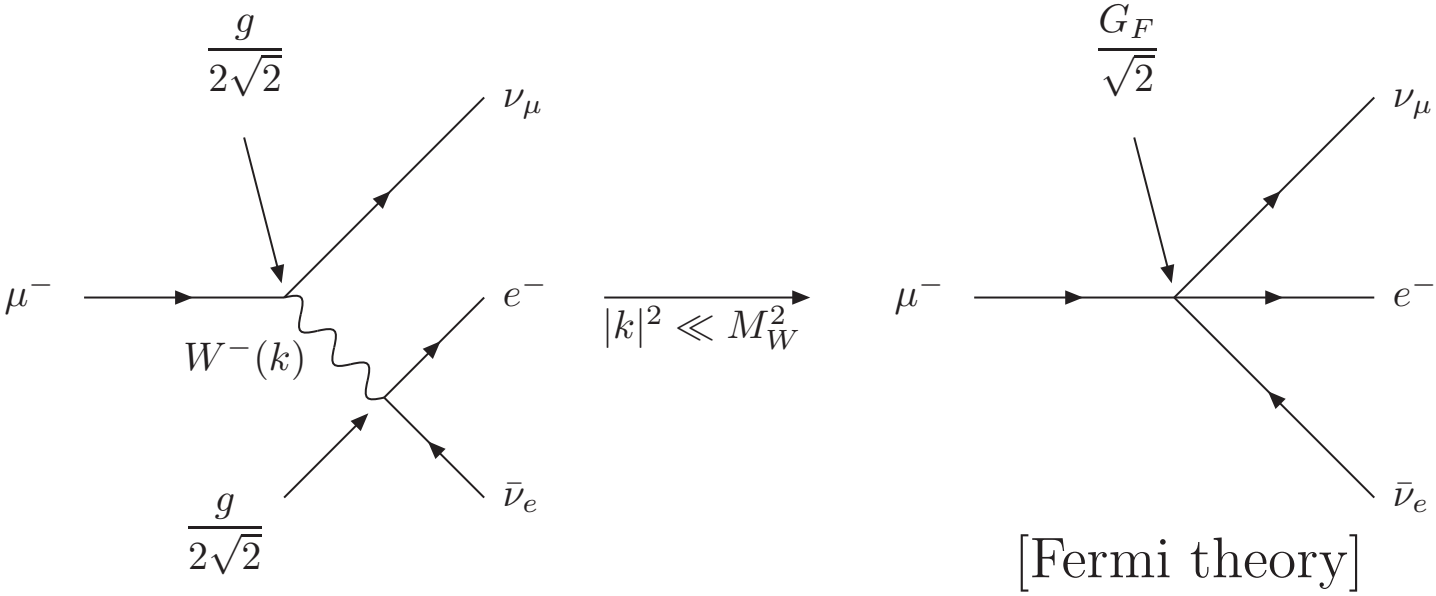
$$-i\frac{M_W}{\sqrt{2}v}\gamma^\mu(1-\gamma_5)$$



$$-i\frac{M_Z}{2v}\gamma^\mu(v_f - a_f\gamma_5)$$

$$\begin{aligned} v_f &= 2I_{3L}^f - 4e_f \sin^2\theta_W \\ a_f &= 2I_{3L}^f \end{aligned}$$

Myon decay:



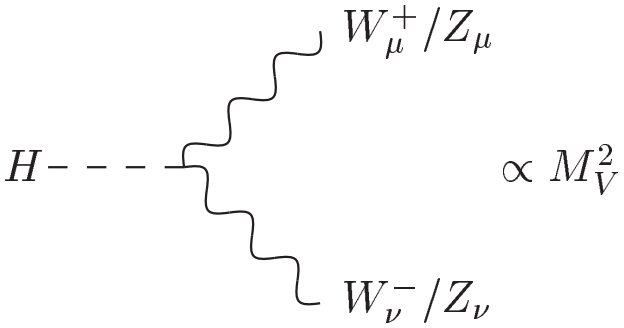
[Fermi theory]

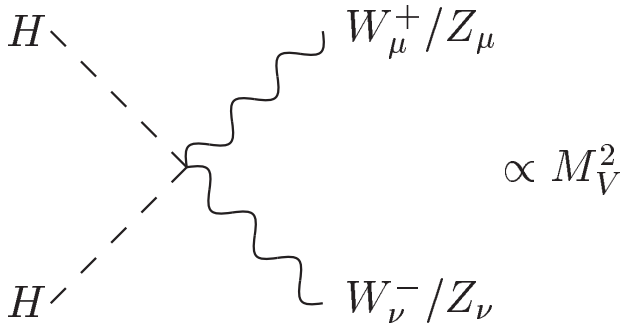
$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{\pi\alpha}{2M_W^2 \sin^2\theta_W}}$$

$$G_F = 1.1663788(7) \cdot 10^{-5}\ \text{GeV}^{-2}$$

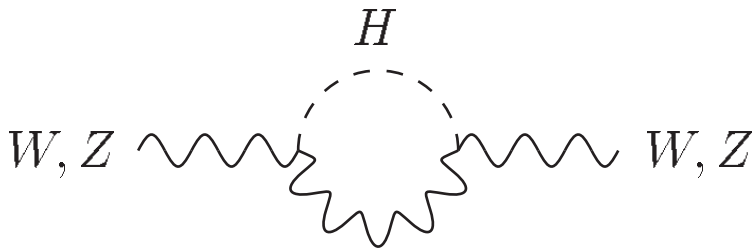
$$v = \frac{2M_W}{g} = \frac{1}{\sqrt{\sqrt{2}G_F}} \approx 246\ \text{GeV}$$

Higgs: $M_H^2 = \lambda v^2$



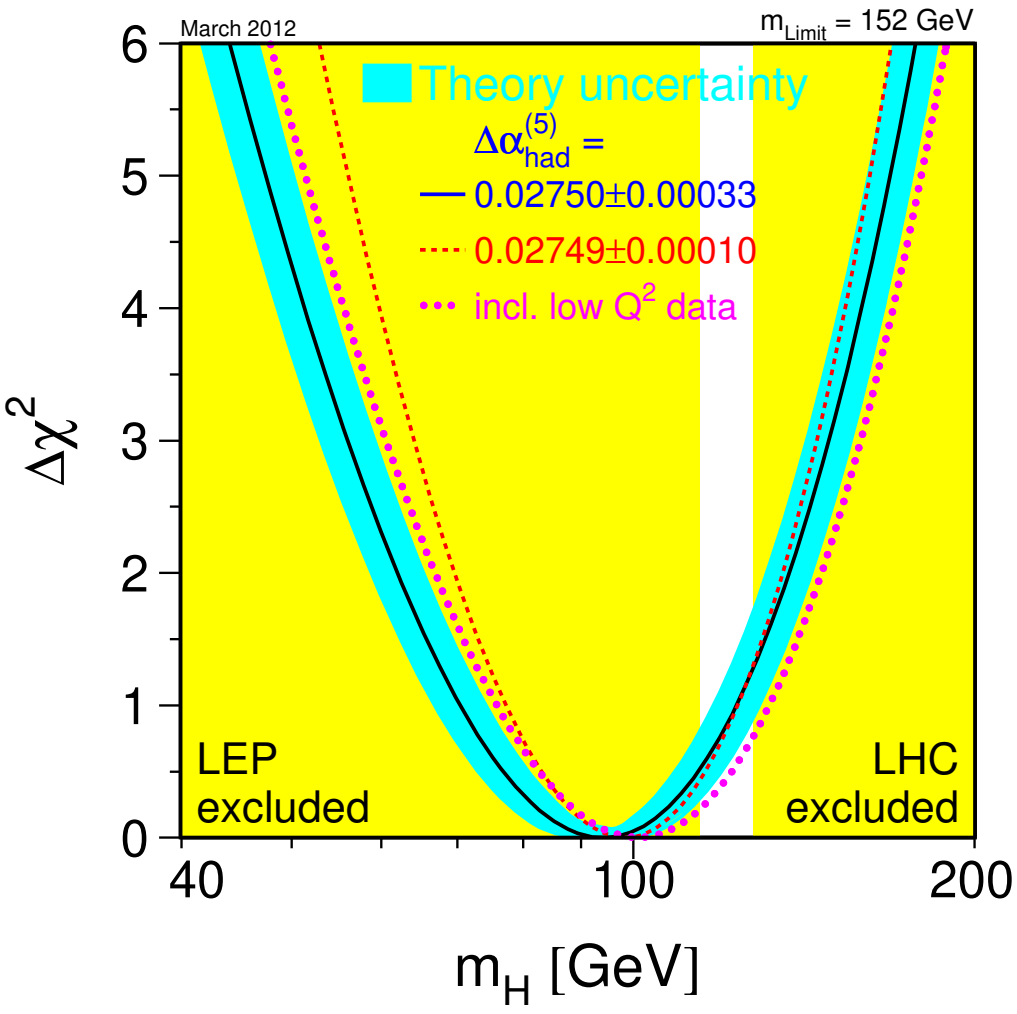


- electroweak observables:



$$G_F = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2\theta_W}[1+\Delta r]$$

$$\Delta r_{Higgs} = \frac{G_F M_W^2}{8\sqrt{2}\pi^2}\,\frac{11}{3}\left[\log\frac{M_H^2}{M_W^2} - \frac{5}{6}\right]$$



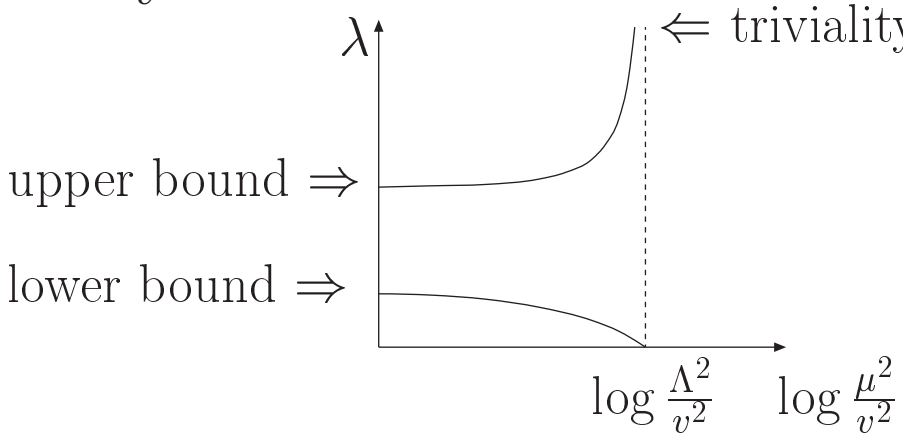
$$M_H = 94^{+29}_{-24}\ \text{GeV}$$

$$\Rightarrow\ M_H \lesssim 152\ \text{GeV (95\% CL)}$$

LEP/SLC

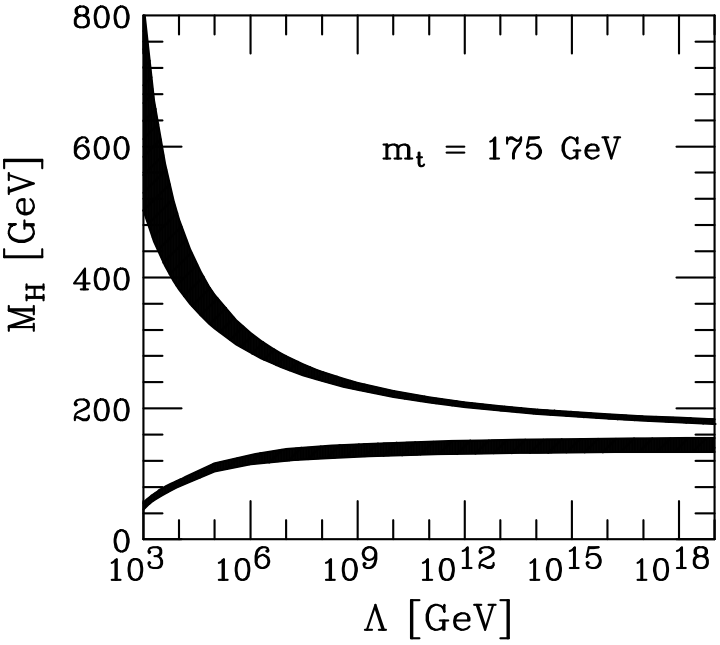
- triviality and vacuum stability:

$$\frac{d\lambda}{d\log\frac{\mu^2}{v^2}} = \frac{3}{8\pi^2}\left[+\lambda^2 + \lambda g_t^2 - g_t^4\right]$$



$$M_H^2 = \lambda v^2$$

$$\Rightarrow \Lambda \gtrsim 1 \text{ TeV}$$



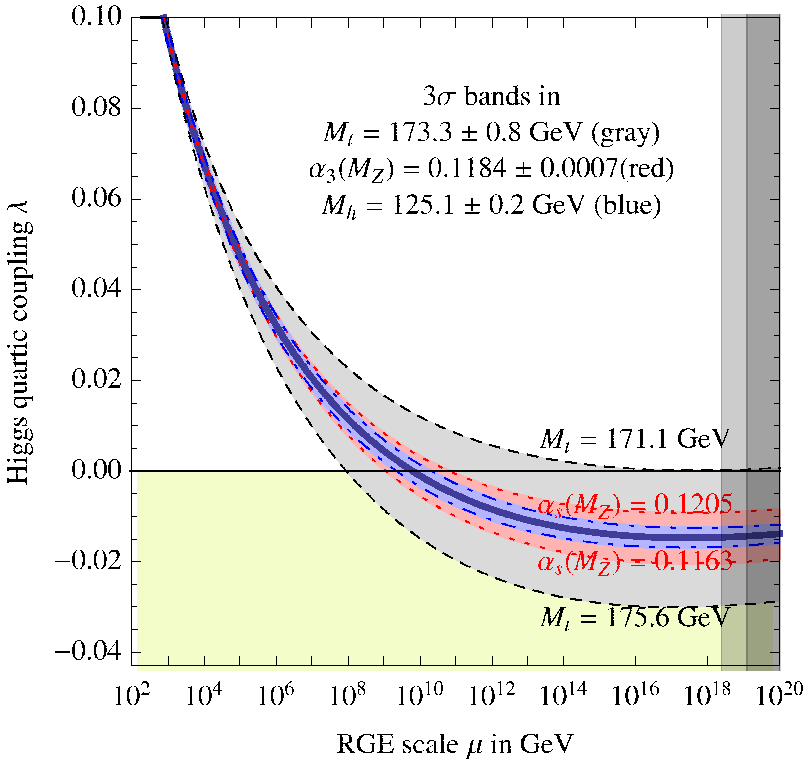
Hambye
Riesselmann

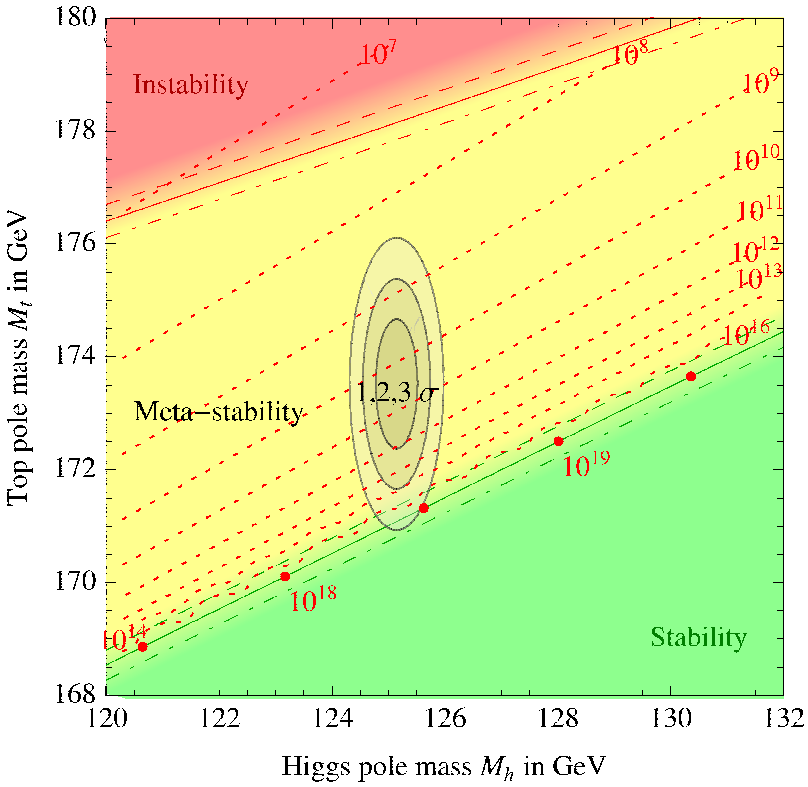
$$\Lambda = 1 \text{ TeV} : 55 \text{ GeV} \lesssim M_H \lesssim 700 \text{ GeV}$$

$$\Lambda = M_{GUT} : 130 \text{ GeV} \lesssim M_H \lesssim 190 \text{ GeV}$$

Cabibbo
Sher
Lindner
Lüscher, Weisz
Hasenfratz,. . .
etc.

$\Rightarrow$ light Higgs boson





Buttazzo, Degrassi,. . .